The matching benefits of market thickness*

Simon Loertscher[†] Ellen V. Muir[‡]

This version: March 24, 2023 First version: February 12, 2021

Abstract

We use an independent private values model to analyze the social benefits and costs of monopoly market makers. Calling products *niche* (*mass*) if the fraction of agents who trade in a Walrasian market is *small* (*large*), we show that for sufficiently niche products a thick market monopoly generates more consumer (producer) surplus per buyer (seller) than ex post efficient bilateral trade. Moreover, relative to bilateral trade, the matching benefits of thick markets grow unboundedly for increasingly niche products. If bilateral trade offers an outside option to trading with a thick market monopoly, mass products better mitigate the monopoly's market power, suggesting a role for regulatory scrutiny in markets for niche products.

Keywords: bilateral trade, Walrasian markets, thick market monopoly, competing exchanges
JEL-Classification: D47, D82, L12

*We thank two referees and an associate editor of this journal for comments that have helped us improve the paper. The paper has also benefited from comments by Gabriel Carroll, Leslie Marx, Paul Milgrom, Alex Nichifor and seminar participants at the University of Melbourne. Toan Le and Hasini Senadheera provided excellent research assistance. Financial support through a Visiting Research Scholar grant from the Faculty of Business and Economics at the University of Melbourne, ARC DP Grant DP200103574 and the Samuel and June Hordern Endowment is also gratefully acknowledged.

†Department of Economics, Level 4, FBE Building, 111 Barry Street, University of Melbourne, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

‡Department of Economics, Stanford University. Email: evmuir@stanford.edu.

1 Introduction

The emergence of online technology giants has fueled public policy debates concerning the appropriate regulation of large monopoly firms. Digital storefronts have enabled online platforms to operate at a scale that is orders of magnitude larger than traditional brick-and-mortar firms.[1] Analyzing the social costs and benefits of these large platforms requires an understanding of the economic forces that govern their size and market power.

In this paper we introduce a simple independent private values model that provides a microfoundation for the social costs and benefits of monopoly market makers, where we distinguish between both the thickness of markets and the nature of products. Thin markets are modelled in terms of a bilateral trade problem, while thick markets involve a continuum of buyers and sellers whose values and costs have the same distributions as those from the bilateral trade problem. The extent to which a product is niche is captured by truncating the distributions from which buyers and sellers draw their types. As products become increasingly niche both the probability that agents can feasibly trade in the bilateral trade problem and the quantity traded in a thick Walrasian market decrease. This model allows us to parameterize the increasing returns to scale in market making by the extent of the double coincidence of wants problem in the bilateral trade setting. Naturally, the double coincidence of wants problem is alleviated in the thick market setting. We show that for sufficiently niche products, a profit-maximizing monopoly operating a thick market generates more consumer surplus per buyer and producer surplus per seller than ex post efficient bilateral trade. We also consider the ratios of per buyer consumer surplus and per seller producer surplus to first-best social surplus in the bilateral trade problem. We show that these ratios diverge to positive infinity as products become perfectly niche.

We also establish two related invariance results. We show that the severity of the incentive problem in bilateral trade, which is captured by the ratio of second-best social surplus divided by first-best social surplus, is independent of the nature of the product. Likewise, we show that, relative to an efficient thick market, the harm from a thick market monopoly does not vary with how niche or mass a product is. The latter implies that the rationale for or against government intervention in a thick monopoly market does not depend on the nature of the product per se. Moreover, it shows that when products are sufficiently niche the benefits of thick market monopolies relative to thin markets for consumer and producer surplus are first order, while their harm relative to efficient thick markets is second order.

That said, there is a sense in which thick market monopolies may be more harmful for

[1] For example, in the early 2000s the number of book titles available on Amazon was twenty times that of a typical, traditional book store (see Brynjolfsson et al., 2003; Waldfogel, 2017).

niche products than for mass products, relative to efficient thick markets. Assuming that competing bilateral exchanges offer agents an outside option to trading with the thick market monopoly, we show in an extension that these competing exchanges are more effective at curbing the market power of a thick market monopoly the less niche a product is. The intuition is simply that the double coincidence of wants problem becomes more severe of an obstacle as products becomes more niche. Therefore, the bilateral exchange becomes a less effective competitor the more niche a product is.

Oliver Williamson made the important observation that larger firms outperform smaller ones because they can always replicate what the smaller ones do, and sometimes do better by *intervening selectively* (Williamson, 1985). The same logic applies to markets because thicker markets can always execute the same trades as smaller, standalone markets and can sometimes execute additional or more valuable trades. This gives rise to increasing returns to scale even when the market maker's technology exhibits constant marginal costs per trade.

Despite its relevance, the ability of thick markets to overcome the double coincidence of wants problem has received relatively scant attention in the mechanism design and the related double-auctions literature.[2] The latter focuses on incentives and has established that the incentive cost of thin markets vanishes quickly as market size increases; see, for example, Gresik and Satterthwaite (1989), Satterthwaite and Williams (1989), McAfee (1992), Rustichini et al. (1994), Satterthwaite and Williams (2002), Tatur (2005), Cripps and Swinkels (2006), Kojima and Yamashita (2017) and Loertscher and Mezzetti (2021). This shows that, as far as incentives are concerned, the benefits from increasing market thickness are small to none. Without necessarily being framed in this way, this literature provides one possible justification for the aggressive approach taken by antitrust authorities of numerous countries to technology giants such as Amazon, Apple, Google, Microsoft and Facebook, whose size and behaviour are perceived as anticompetitive and as an abuse of market power. Our paper contributes to these ongoing debates by distilling what may be an issue of first-order importance: the benefits of market thickness and the increasing returns to scale in market making that arise for niche products. According to Waldfogel (2017), digitalization has led to a "golden age of music, movies, books and television." Anderson (2006) provides an early account of the benefits that accrue to consumers (and producers) from the access to the "long tail" that digitalization grants. He notes that "Netflix changed the economics of niches, and in so doing, reshaped our understanding about what people actually *want* to watch" and reports that for books, "Barnes & Noble found that the bottom 1.2 million titles represent

[2]One notable exception is Bulow and Klemperer (1996) who demonstrate the importance of increasing the number of buyers in one-sided settings by showing that a second-price auction with no reserve price but one additional bidder outperforms the optimal auction.

just 1.7 percent of its instore sales, but a full 10 percent of its online (bn.com) sales."[3]

Some of the key issues this paper brings to light are also present in Loertscher et al. (2022), but are somewhat camouflaged there by the dynamic setting. By studying a one-shot setup, the present paper provides a clean-cut and transparent analysis of the benefits of thicker markets, and the relative importance of incentives and improved matching in thicker markets. Moreover, in the setting considered in Loertscher et al. (2022), provided a non-trivial dynamic market-clearing mechanism is optimal (i.e. provided it is not optimal for the designer to simply execute a static bilateral trade in each period), there is no incentive problem and all of the benefits from increasing market thickness stem from improved matching of traders. Part of the purpose of this paper is to show that large matching benefits from increasing market thickness can also arise in static markets. To the best of our knowledge, our paper is the first to use the truncation-invariance of virtual type functions to model the benefits of market thickness.

Of course, market thickness and thinness also play an important role in the literature on financial markets, where they are typically related to an agent's price impact in a continuous-time double auction (see e.g. Rostek and Weretka, 2008). In contrast, the definitions used here (and in Loertscher et al., 2022) are independent of the trading mechanism, implying that a market can be thin even if a dominant-strategy mechanism—such as the one from Hagerty and Rogerson (1987)—is used.

Our extension in which bilateral trade offers an outside option to trading with a monopoly draws inspiration from the literature on intermediaries facing a competing exchange; see Gehrig (1993), Spulber (2002), Rust and Hall (2003), and Loertscher and Niedermayer (2020). An important departure of our model from the approach in this literature is that we assume that the competing exchanges involve bilateral trade, which implies that the value of the outside option for agents depends on whether products are niche or mass. In contrast, in the aforementioned papers, the competing exchanges are thick markets.

The remainder of this paper is organized as follows. Section 2 introduces the setup. In Section 3 we derive our results concerning thick Walrasian markets and thick market monopolies and relate them to properties of the bilateral trade problem. Section 4 discusses returns to scale in market making for niche products, as well as the implications of competing bilateral exchanges for market power in markets for niche products. Section 5 concludes the paper. All proofs are provided in Appendix A and additional extensions and robustness checks can be found in Appendix B.

[3] Brynjolfsson et al. (2003) estimate that Amazon's book sales for titles ranked outside the top 100,000 accounted for 39.2% of total book sales of Amazon.

2 Setup

Throughout this paper we consider static market settings involving risk-neutral buyers and sellers with unit demand and unit capacity, respectively, whose utility functions are quasi-linear. We assume that each buyer is privately informed about the realization of its value v and each seller is privately informed about its realized cost c. Under these assumptions, optimal mechanisms are well-defined without restrictions on the contracting space. With the exception of the analysis in Section 4.2, the value of the outside option of not participating in the market is 0 for every agent.

We first introduce the notion of a *mass product*. In markets for a mass product buyers and sellers draw their types from distributions with a common support. Specifically, buyer values $v \in [0,1]$ are distributed according to the distribution F with density f that has full support on $[0,1]$. Similarly, seller costs $c \in [0,1]$ are distributed according to the distribution G with density g that has full support on $[0,1]$. For any $\alpha \in [0,1]$, the *weighted virtual type functions* are then

$$\Phi_\alpha(v) := v - \alpha\frac{1-F(v)}{f(v)} \quad \text{and} \quad \Gamma_\alpha(c) := c + \alpha\frac{G(c)}{g(c)}.$$

We also let $\Phi(v) := \Phi_1(v)$ and $\Gamma(c) := \Gamma_1(c)$ denote the *virtual type functions* that have weight of $\alpha = 1$ on revenue. To develop an intuitive understanding of these functions, consider first Φ and Γ. As observed by Mussa and Rosen (1978) and Bulow and Roberts (1989), these functions can be interpreted as marginal revenue and marginal (procurement) cost functions. To see this, notice first that $F^{-1}(1-q)$ and $G^{-1}(q)$ can be viewed as the inverse demand and supply functions given quantity $q \in [0,1]$, implying that revenue and procurement cost as a functions of q are $H_F(q) := qF^{-1}(1-q)$ and $H_G(q) := qG^{-1}(q)$. Consequently, marginal revenue and marginal cost are $H_F'(q)|_{p=F^{-1}(1-q)} = \Phi(p)$ and $H_G'(q)|_{p=G^{-1}(q)} = \Gamma(p)$. Notice next that $\Phi_\alpha(v) = (1-\alpha)v + \alpha\Phi(v)$ and $\Gamma_\alpha(c) = (1-\alpha)c + \alpha\Gamma(c)$. That is, the weighted virtual type functions are convex combinations of the true types v and c and the virtual types $\Phi(v)$ and $\Gamma(c)$, with weight α on the virtual types. Consequently, the weighted virtual type functions $\Phi_\alpha(v)$ and $\Gamma_\alpha(c)$ are the analogues to the marginal revenue and marginal cost functions derived from a Ramsey pricing problem in which the regulator's weight on revenue is α.[4] For any $\alpha \in [0,1]$, we let $\overline{\Phi}_\alpha(v)$ and $\overline{\Gamma}_\alpha(c)$ respectively denote the weighted *ironed* virtual valuation and cost functions.[5] As we just saw, Φ and Γ represent marginal revenue and

[4]For more on Ramsey pricing, see, for example, Wilson (1993). Bulow and Roberts (1989) make the connection between the weighted virtual type functions used in mechanism design and Ramsey pricing while Myerson and Satterthwaite (1983) show that $\alpha = \lambda/(1+\lambda)$ in the bilateral trade problem they study, where $\lambda > 0$ is the solution value of the Lagrange multiplier associated with the no-deficit constraint.

[5]The ironed virtual type functions are the marginal revenue and marginal cost functions associated

marginal cost under *market-clearing* pricing. However, in cases where one of these functions is not monotone (and market-clearing pricing is not necessarily optimal for both sides of the market) the ironed functions $\overline{\Phi}$ and $\overline{\Gamma}$ represent marginal revenue and marginal cost under the *optimal* mechanism.

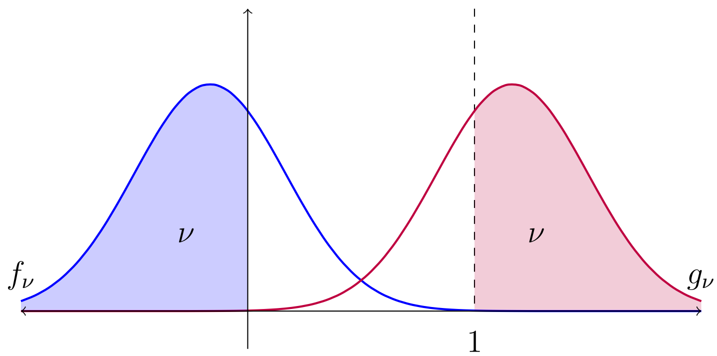


Figure 1: An illustration of the density functions f_ν and g_ν for a niche market.

In the market for a *niche product* with parameter $\nu \in [0, 1)$ we assume that buyer values $v \in [a_\nu, 1]$ with $a_\nu < 0$ are drawn from some distribution F_ν with density f_ν that has full support on $[a_\nu, 1]$ and seller costs $c \in [0, b_\nu]$ with $b_\nu > 1$ are drawn from some distribution G_ν with density g_ν that has full support on $[0, b_\nu]$. While these distributions may be arbitrary outside the interval $[0, 1]$ of overlapping support, on this interval these distributions are restricted so that for all $\nu \in [0, 1)$, $v \in [0, 1]$ and $c \in [0, 1]$, we have

$$F_\nu(0) = 1 - G_\nu(1) = \nu, \quad \frac{F_\nu(v) - \nu}{1 - \nu} = F(v) \quad \text{and} \quad \frac{G_\nu(c)}{1 - \nu} = G(c).$$

That is, in a market for a niche product with parameter $\nu \in [0, 1)$, buyers have a value $v \in [a_\nu, 0)$ with probability $\nu \in [0, 1)$ and a value $v \in [0, 1]$ with probability $1-\nu$. Conditional on $v \in [0, 1]$, buyers values are distributed according to F. Similarly, sellers have a cost $c \in (1, b_\nu]$ with probability ν and a cost $c \in [0, 1]$ with probability $1 - \nu$. Conditional on $c \in [0, 1]$, sellers costs are distributed according to G. See Figure 1 for an illustration. The following observation will play an important role in our analysis. We state it as a lemma since it seems a useful fact without claiming or believing that we are the first to make this

with the concavification of the revenue function and the convexification of the procurement cost function, respectively; see, for example, Myerson (1981). Specifically, we let $\overline{H}_F$ denote the concavification of H_F and $\overline{H}_G$ denote the convexification of H_G. The ironed virtual type functions are then given by $\overline{\Phi}(v) = \overline{H}'_F(q)|_{q=1-F(v)}$ and $\overline{\Gamma}(c) = \overline{H}'_G(q)|_{q=G(c)}$, while the weighted ironed virtual type functions are given by $\overline{\Phi}_\alpha(v) = (1-\alpha)v + \alpha\overline{\Phi}(v)$ and $\overline{\Gamma}_\alpha(c) = (1-\alpha)c + \alpha\overline{\Gamma}(c)$. Here, the concavification (also known as the upper concave envelope) of a given function is the smallest concave function that is weakly larger than that function at every point in its domain. The convexification (or lower convex envelope) of a given function is the largest convex function that is weakly less than that function at every point in its domain.

observation:

Lemma 1. *The weighted virtual type functions are truncation invariant. That is, for any* $\alpha \in [0,1]$, $\nu \in [0,1]$, $v \in [0,1)$ *and* $c \in [0,1]$, *we have*

$$\Phi_\alpha(v) = v - \alpha \frac{1 - F_\nu(v)}{f_\nu(v)} \quad \textit{and} \quad \Gamma_\alpha(c) = c + \alpha \frac{G_\nu(c)}{g_\nu(c)}.$$

We study two extreme cases that encompass both perfectly thin and perfectly thick markets. On the *thin* end of the spectrum we consider the *bilateral trade problem* of Myerson and Satterthwaite (1983) which involves exactly one buyer and one seller. While some of our results—such as Proposition 1—hold for any α, our benchmark for the bilateral trade problem is social surplus under the *second-best* mechanism, which maximizes expected gains from trade subject to agents' incentive compatibility and interim individual rationality constraints and a no-deficit constraint for the designer.[6] By construction, the designer's expected profit under the second-best mechanism is 0. In contrast to the second-best mechanism, the allocation rule under the *first-best* mechanism is to induce trade whenever $v \geq c$. On the *thick* end of the spectrum we consider markets involving a continuum of traders, with an equal mass of buyers and sellers on each side of the market. In this case we look at both the *thick Walrasian market*, in which there is a benevolent (arguably fictitious) auctioneer who organizes the exchange and quotes the market clearing price, and the *thick market monopoly*, where the thick market is organized by a profit-maximizing monopoly that has zero costs.[7] In either case, the mechanism consists of posting price(s) since the market maker faces no aggregate uncertainty. Consequently, the agents act as price-takers in large markets.

The setting introduced here provides a sharp illustration of the main insights of this paper. However, the results in Section 3 can be generalized to allow for asymmetric niche product parameters for buyers and sellers (see Appendix B.2) and to allow for the region of overlapping support between the type distributions to vary with ν (see Appendix B.3). Our setup is general insofar as we impose only mild restrictions on the distributions from which the agents draw their their types. Its tractability derives from the way the niche nature of products is modelled, which is done with a single parameter, without imposing any parameteric restrictions on the type distributions.

[6]Formally, the second-best mechanism maximizes $\mathbb{E}[(v-c)Q(v,c)]$ over $Q : [0,1]^2 \to [0,1]$ subject to the aforementioned constraints. The solution to this problem is given by the allocation rule $Q^{\alpha^*}(v,c)$ that induces trade if and only if $\overline{\Phi}_{\alpha^*}(v) \geq \overline{\Gamma}_{\alpha^*}(c)$, where $\alpha^* \in (0,1)$ is the unique number such that the expected revenue of the designer is 0. That is, such that $\mathbb{E}[(\Phi(v) - \Gamma(c))Q^{\alpha^*}(v,c)] = 0$.

[7]See Aumann (1964) for a formalization of competitive equilibrium and its relation to the core with a continuum of traders. Implicitly or explicitly, the theory of monopoly and oligopoly pricing dating back to Cournot (1838) has rested on the assumption of a continuum of price-taking agents on sides of the market without power.

Of course, niche products can naturally arise in many alternative settings. For example, one can consider type distributions with unbounded support, where products become increasingly niche as probability mass is shifted towards the tails of the type distributions in an appropriate fashion (see Appendix B.4). One can also model niche products by parameterizing the type distributions so that the elasticity of demand and supply increases and the Walrasian quantity traded decreases as products become increasingly niche (see Appendix B.5). As shown there, the main insights carry over to these alternative specifications.

3 Analysis

For $\nu \in [0,1)$, let $S_1^{FB}(\nu)$ and $S_1^{SB}(\nu)$ denote first-best and second-best welfare, respectively, in the bilateral trade setting.[8] Similarly, we let $S_\infty(\nu)$ denote welfare per trader pair in the thick Walrasian market. In this section we analyze the thick Walrasian market and the thick market monopoly by relating them to the bilateral trade problem. In particular, we will be interested in studying how $S_1^{FB}(\nu), S_1^{SB}(\nu)$ and $S_\infty(\nu)$ and their ratios vary with ν. The mass product ratios

$$s_1 := \frac{S_1^{SB}(0)}{S_1^{FB}(0)} \quad \text{and} \quad s_\infty := \frac{S_\infty(0)}{S_1^{FB}(0)}$$

with $0 < s_1 < 1 < s_\infty$ provide a natural benchmark.[9] We will also study how consumer surplus per buyer, producer surplus per seller, and social surplus per trader pair vary with ν and how they compare to $S_1^{FB}(\nu)$ when the thick market is organized by a profit-maximizing monopoly.

3.1 Thin markets

We begin by analyzing the bilateral trade problem. As noted, conditional on having both $v \in [0,1]$ and $c \in [0,1]$, agents' type distributions (and hence agents' virtual type functions) do not vary with the parameter ν. Consequently, the first-best mechanism and second-best mechanisms do not depend on ν and we have

$$S_1^{FB}(\nu) = (1-\nu)^2 S_1^{FB}(0) \quad \text{and} \quad S_1^{SB}(\nu) = (1-\nu)^2 S_1^{SB}(0), \tag{1}$$

[8] As shown by Myerson and Satterthwaite (1983), first-best trade is not possible when the agents are privately informed about their values and costs, their interim incentive compatibility and individual rationality constraints have to be satisfied and the mechanism must not run a deficit. Moreover, by assumption, full trade is not optimal in this setting.

[9] The impossibility theorem of Myerson and Satterthwaite (1983) implies $s_1 < 1$. Because our assumptions imply that full trade is not optimal in this setting, we also have $s_\infty > 1$.

where $(1-\nu)^2$ is the probability that $v \in [0,1]$ and $c \in [0,1]$. More generally, for any $\alpha \in [0,1]$ we can consider the mechanism that maximizes a convex combination of social welfare and designer profit, where α is the weight on the designer's profit. Letting $S_1^\alpha(\nu)$ denote welfare under this mechanism, we have the following proposition.

Proposition 1. *For any $\alpha \in [0,1]$ and $\nu \in [0,1)$, we have $S_1^\alpha(\nu) = (1-\nu)^2 S_1^\alpha(0)$, where $S_1^\alpha(0) = \mathbb{E}[(v-c)Q^\alpha(v,c)]$ and $Q^\alpha(v,c) = \mathbb{1}(\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c))$. Consequently, the ratio $\frac{S_1^\alpha(\nu)}{S_1^{FB}(\nu)}$ does not vary with ν.*

The class of mechanisms considered in Proposition 1 is equivalent to the class of mechanisms that implement Ramsey pricing.[10] This class of mechanisms traces out a frontier characterizing the optimal tradeoff between revenue and welfare, encompassing welfare maximization (the first-best mechanism), welfare maximization subject to budget balance (the second-best mechanism) and profit maximization.

In some applications, other mechanisms, such as the trade sacrifice mechanism of McAfee (1992), may be more practical. With that in mind, we have the following extension of Proposition 1 to the posted-price mechanism of Hagerty and Rogerson (1987), which is what the trade sacrifice mechanism of McAfee (1992) specializes to when there is only one buyer and one seller. Letting $S_1^{HR}(\nu)$ denote Hagerty-Rogerson welfare in the bilateral trade problem we have

$$S_1^{HR}(\nu) = (1-\nu)^2(1-F(p))G(p)\mathbb{E}[v-c|v \geq p, c \leq p] = (1-\nu)^2 S_1^{HR}(0),$$

showing that the ratio $\frac{S_1^{HR}(\nu)}{S_1^{FB}(\nu)}$ does not vary with ν.

3.2 Thick Walrasian markets

We now analyze thick markets involving a continuum of traders, starting with thick Walrasian markets. Analogously to the case involving bilateral trade, the market clearing price p is independent of ν and is such that $1-F(p) = G(p)$. Letting $w = 1-F(p) = G(p)$ denote the *Walrasian quantity* in a market for mass products (strictly speaking, the probability that any given agent trades in an efficient thick market, conditional on its type being between 0 and 1), the Walrasian quantity in a market for niche products is given by $w(1-\nu)$. Welfare

[10] Under Ramsey pricing social welfare is maximized subject to the constraint that the mechanism generates profit of at least k. The second-best mechanism corresponds to Ramsey pricing with $k=0$. This class of mechanisms is also equivalent to the class of mechanisms that maximize profit subject to the constraint that some minimal level of welfare is achieved. See Wilson (1993) and Bulow and Roberts (1989).

per trader pair in the thick Walrasian market is then

$$S_\infty(\nu) = w(1-\nu)\mathbb{E}[v-c|v \geq p, c \leq p] = (1-\nu)S_\infty(0).$$

This setup and the role of the niche product parameter ν in thick Walrasian markets is illustrated in Figure 2.

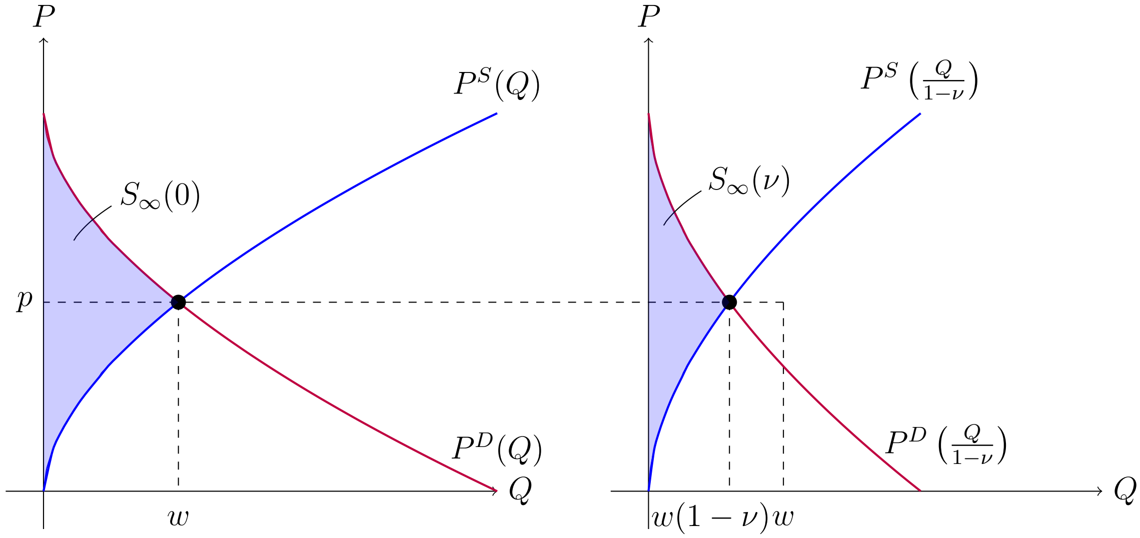


Figure 2: The thick Walrasian market for a mass product and for a niche product.

Combining all of this with our analysis from the previous section, we have the following proposition.

Proposition 2. *For any $\nu \in [0,1)$ we have $\frac{S_\infty(\nu)}{S_1^{FB}(\nu)} = \frac{s_\infty}{1-\nu} > 1$ and hence $\lim_{\nu\to 1}\frac{S_\infty(\nu)}{S_1^{FB}(\nu)} = \infty$.*

Propositions 1 and 2 provide a formalization of the importance of avoiding the double coincidence of wants problem for niche products. As is well known from the double auctions literature, the incentive cost of small markets vanishes quickly as the size of a market increases. Since the second-best mechanism arises as a special case of Proposition 1, this proposition shows that the ratio $\frac{S_1^{SB}(\nu)}{S_1^{FB}(\nu)}$, which provides an upper bound on the incentive cost of small markets, does not depend on the niche market parameter ν. In contrast, the relative benefits of thick markets, captured by the ratio $\frac{S_\infty(\nu)}{S_1^{FB}(\nu)}$, increase in ν at rate $(1-\nu)^{-1}$. Intuitively, thin niche markets suffer from a severe double coincidence of wants problem. First-best welfare in the bilateral trade setting is proportional to $(1-\nu)^2$ because any given agent with a type that lies within the interval $[0,1]$ is only matched with another such agent with probability $1-\nu$. In the thick Walrasian market welfare per trader pair is proportional to $1-\nu$ and any buyer with value $v \geq p$ and any seller with cost $c \leq p$ trades with probability 1.

3.3 Thick market monopoly

We now consider a thick market with niche product parameter ν that is operated by a monopoly market maker. We let $CS_\infty^M(\nu)$, $PS_\infty^M(\nu)$ and $S_\infty^M(\nu)$ denote consumer surplus per buyer, producer surplus per seller and total surplus per trader pair, respectively. In this case it is useful to distinguish whether or not the functions Φ and Γ are increasing. When these functions are increasing, customarily referred to as the *regular* case, this implies that the marginal revenue and marginal cost functions facing the monopolist are decreasing and increasing, respectively. The optimal mechanism under a thick market monopoly then involves posting prices p_B (for buyers) and p_S (for sellers) on both sides of the market. These prices equate marginal revenue and marginal cost and balance the quantity traded. That is, they satisfy

$$\Phi(p_B) = \Gamma(p_S) \quad \text{and} \quad 1 - F(p_B) = G(p_S). \tag{2}$$

To see this, notice that if the monopoly trades the quantity $q \in [0, 1]$ at market clearing prices $p_B = F^{-1}(1 - q)$ and $p_S = G^{-1}(q)$, its profit maximization problem is

$$\max_{q\in[0,1]} (F^{-1}(1 - q) - G^{-1}(q))q, \tag{3}$$

which yields the first-order condition

$$\Phi(F^{-1}(1 - q)) - \Gamma(G^{-1}(q)) = \Phi(p_B) - \Gamma(p_S) = 0.$$

The second-order condition is satisfied if $\Phi(v)$ and $\Gamma(c)$ are increasing, proving that the first-order condition characterizes a maximum and hence that the prices in (2) are optimal and unique. Moreover, they are independent of ν and such that $0 < p_S < p_B < 1$. Let $m = G(p_S) = 1 - F(p_B)$ denote the probability that any given buyer or seller trades under the thick market monopoly, conditional on having a value or cost between 0 and 1. The unconditional probability of trading is then $m(1 - \nu)$. Accordingly, in the regular case, consumer surplus per buyer and producer surplus per seller under the thick market monopoly are

$$CS_\infty^M(\nu) = m(1 - \nu)\mathbb{E}[v - p_B|v \geq p_B] \quad \text{and} \quad PS_\infty^M(\nu) = m(1 - \nu)\mathbb{E}[p_S - c|c \leq p_S].$$

Moreover, the monopoly's profit per trader pair is $m(1-\nu)(p_B - p_S)$ and consequently total surplus per trader pair is given by

$$S_\infty^M(\nu) = CS_\infty^M(\nu) + PS_\infty^M(\nu) + m(1 - \nu)(p_B - p_S).$$

More generally, if the type distributions are not restricted to be regular, we can apply the analysis of Loertscher and Muir (2022) to obtain similar expressions for consumer surplus, producer surplus and the monopoly's profit per trader pair.

An interesting and important question is whether, relative to a thick Walrasian market, a thick market monopoly is more harmful for consumers and producers with niche or with mass products. As we show next, the harm from monopolies that intermediate a thick market does not depend on the nature of the product. Letting $CS_\infty(\nu)$ and $PS_\infty(\nu)$ respectively denote consumer surplus per buyer and producer surplus per seller in a thick Walrasian market we have

$$CS_\infty(\nu) = w(1-\nu)\mathbb{E}[v-p|v \geq p] \quad \text{and} \quad PS_\infty(\nu) = w(1-\nu)\mathbb{E}[p-c|c \leq p].$$

Putting all of this together yields the following proposition.

Proposition 3. *For the thick market monopoly, there exists a $\hat{\nu} < 1$ such that for all $\nu > \hat{\nu}$,*

$$CS_\infty^M(\nu) > S_1^{FB}(\nu) \quad \textit{and} \quad PS_\infty^M(\nu) > S_1^{FB}(\nu).$$

Moreover, $\lim_{\nu\to 1} \frac{CS_\infty^M(\nu)}{S_1^{FB}(\nu)} = \lim_{\nu\to 1} \frac{PS_\infty^M(\nu)}{S_1^{FB}(\nu)} = \infty$ *and the ratios* $\frac{CS_\infty^M(\nu)}{CS_\infty(\nu)}$, $\frac{PS_\infty^M(\nu)}{PS_\infty(\nu)}$ *and* $\frac{S_\infty^M(\nu)}{S_\infty(\nu)}$ *are finite and independent of* ν *for* $\nu \in [0,1)$.

In markets for products that are sufficiently "niche" both consumer surplus per buyer and producer surplus per seller under a thick market monopoly exceed total surplus under ex post efficient bilateral trade, and as ν approaches 1, the ratio of consumer surplus per buyer and producer surplus per seller under the thick market monopoly over first-best welfare in bilateral trade diverge to infinity. Of course, because social surplus is larger than the sum of consumer and producer surplus, Proposition 3 implies that social surplus per trader pair under the thick market monopoly over first-best welfare in the bilateral trade problem also diverge to infinity as ν approaches 1.

Proposition 3 also states an invariance result for thick markets: that the severity of monopoly harm does not vary with ν. This parallels the invariance result for bilateral trade as stated in Proposition 3, which shows that the severity of the incentive problem for thin products is independent of the nature of the product. Like Proposition 3, the results stated in Proposition 3 are driven by the truncation invariance of the virtual type functions.

Proposition 3 has two potentially important policy implications. First, it means that the justification for governments to intervene or not to intervene in thick monopoly markets is the same for niche and for mass products.[11] Second, in combination with Proposition 3, it

[11]See, however, Section 4.2, which shows that competing bilateral exchanges are more effective in curbing

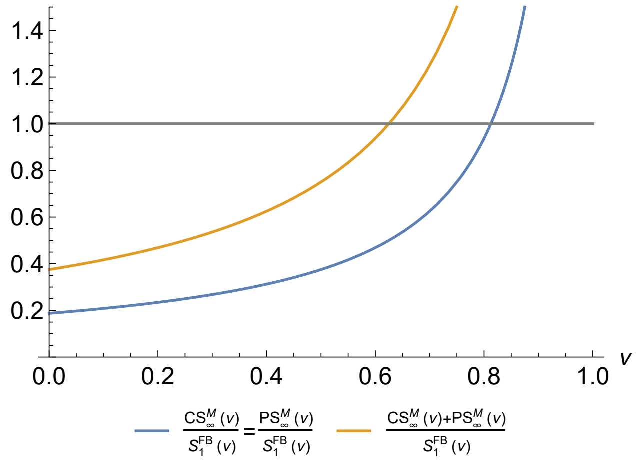


Figure 3: The ratios $CS^M_\infty(\nu)/S^{FB}_1(\nu) = PS^M_\infty(\nu)/S^{FB}_1(\nu)$ (blue) and $(CS^M_\infty(\nu) + PS^M_\infty(\nu))/S^{FB}_1(\nu)$ (orange) as functions of ν for F and G uniform.

implies that for ν sufficiently large, the consumer and producer benefits from thick market monopolies relative to thin markets are first-order, while the harm from monopolies relative to an efficient thick market is second order.

The following example illustrates the results of Proposition 3 when buyer values and seller costs are uniformly distributed.

Example 1. *As an illustration, assume that F and G are uniform. Then $p_B = 3/4$ and $p_S = 1/4$, implying $m = 1/4$ and $CS^M_\infty(\nu) = PS^M_\infty(\nu) = (1-\nu)/32$ while the market maker's profit is $m(1-\nu)(p_B - p_S) = (1-\nu)/8$. We also have $CS_\infty(\nu) = PS_\infty(\nu) = (1-\nu)/8$ and $S_\infty(\nu) = (1-\nu)/4$, which implies that $CS^M_\infty(\nu)/CS_\infty(\nu) = PS^M_\infty(\nu)/PS_\infty(\nu) = 1/4$ and $S^M_\infty(\nu)/S_\infty(\nu) = 3/4$. Moreover, $S^{FB}_1(0) = 1/6$ and, from (1), we have $S^{FB}_1(\nu) = (1-\nu)^2 S^{FB}_1(0)$. Thus, the ratios $CS^M_\infty(\nu)/S^{FB}_1(\nu)$ and $PS^M_\infty(\nu)/S^{FB}_1(\nu)$ are equal to $3/(16(1-\nu))$. As illustrated in Figure 3, this is increasing in ν and larger than 1 for $\nu > 13/16 = 0.8125$. The figure also displays $(CS^M_\infty(\nu)+PS^M_\infty(\nu))/S^{FB}_1(\nu) = 3/(8(1-\nu))$, that is, the sum of consumer surplus per buyer and producer surplus per seller divided by first-best welfare under bilateral trade. This ratio is larger than 1 for $\nu > 5/8 = 0.625$. This shows that the matching benefits of thick monopoly markets come to bear for ν well below 1. Moreover, if one were to adopt a social surplus perspective and also take the monopoly's profit into account, the monopoly would outperform first-best bilateral trade even with mass products since social surplus under the thick market monopoly is $3/16$, which exceeds $S^{FB}_1(0) = 1/6$.*[12]

the market power of thick market monopolies for mass products than for niche products.

[12]This is a point first noted in Loertscher et al. (2022).

4 Discussion

We now provide a brief analysis of the increasing returns to scale from market making and show that markets for niche products exhibit greater returns to scale. We also show that mass products naturally mitigate the market power of monopoly market makers faced with competing bilateral exchanges, whereas niche products do not. This provides a rationale for more regulatory scrutiny for thick market monopolies for niche products than for mass products.

4.1 How big are the returns to scale in market making?

We begin by analysing the returns to scale in market making. Specifically, we now consider markets for niche products involving n pairs of buyers and sellers with independently distributed types. Let $S_n^{FB}(\nu)$ denote welfare per trader pair in a Walrasian market with n pairs of buyers and sellers. A natural measure of market thickness is to consider the proportion of the maximum increase in per trader pair surplus

$$T_n(\nu) := \frac{S_n^{FB}(\nu) - S_1^{FB}(\nu)}{S_\infty(\nu) - S_1^{FB}(\nu)}$$

that is achieved by a market with n pairs of buyers and sellers. Here, $S_\infty(\nu) - S_1^{FB}(\nu)$ captures the maximum achievable increase in welfare per trader pair as we move from a perfectly thin market with a single pair of traders to a perfectly thick market in the limit as $n \to \infty$. This measure of market thickness is analogous to that introduced in Loertscher et al. (2022). Accordingly, $T_{n+1}(\nu)/T_n(\nu)$ provides a natural measure of the returns to scale in market making.

In order to compute $T_n(\nu)$ and $T_{n+1}(\nu)/T_n(\nu)$, we start by computing expected welfare $S_n^{FB}(\nu)$ in a Walrasian market with n pairs of buyers and sellers by exploiting its recursive structure. Specifically, $S_{n+1}^{FB}(\nu)$ is given by $S_n^{FB}(\nu)$ plus the expected increase in welfare associated with adding an additional seller to a market with n buyers and n sellers plus the expected increase in welfare associated with adding an additional buyer to a market with n buyers and $n+1$ sellers. Given a niche market with n buyers and m sellers we let $X = \{v_1, \ldots, v_n, c_1, \ldots, c_m\}$ denote the joint set of trader valuations and costs, $x_{(1)} \leq \cdots \leq x_{(n+m)}$ denote the order statistics of this set and $h_{\nu,i,n,m}(x)$ denote the density of the ith order statistic $x_{(i)}$, that is, the ith lowest draw. We then have the following proposition.

Proposition 4. *Welfare in a Walrasian market with* $n+1$ *pairs of buyers and sellers satisfies*

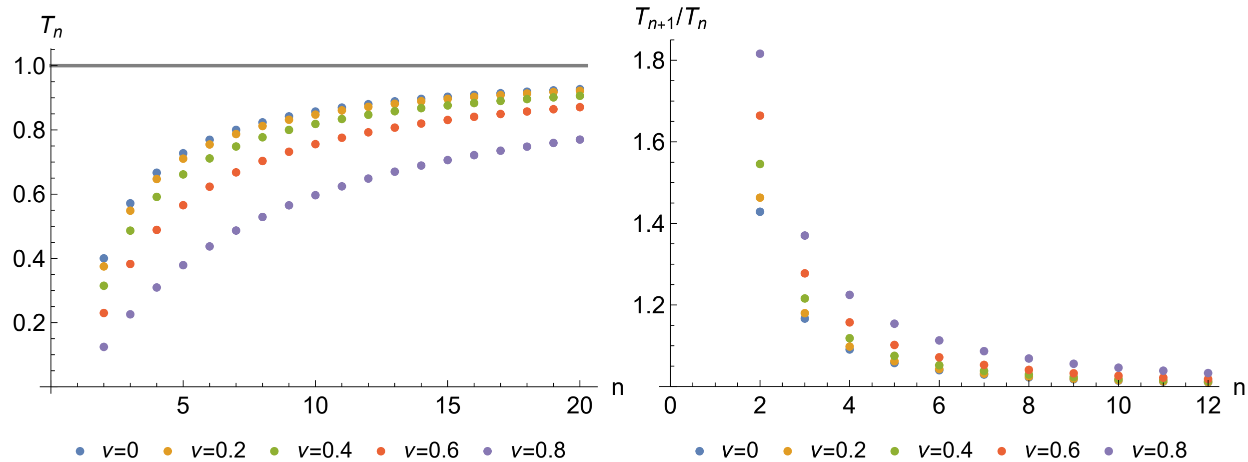


Figure 4: Illustration of our measure of market thickness T_n and returns to scale in market making T_{n+1}/T_n for uniform distributions and various values of ν and n.

the recursion

$$S_{n+1}^{FB}(\nu) = S_n^{FB}(\nu) + (1-\nu)\int_0^1 \int_0^x (x-c)g(c)h_{\nu,n,n,n}(x)\,dc\,dx$$
$$+ (1-\nu)\int_0^1 \int_x^1 (v-x)f(v)h_{\nu,n+1,n,n+1}(x)\,dv\,dx,$$

with $S_1^{FB}(\nu) = (1-\nu)^2 \int_0^1 \int_c^1 (v-c)f(v)g(c)\,dv\,dc$*.*

Figure 4 provides an illustration of the returns to scale in market making, $\frac{T_{n+1}(\nu)}{T_n(\nu)}$, for the special case of uniform distributions. Corollary B.1 in Appendix B.7 provides closed-form solutions for the expressions plotted here. The figure shows that the returns to scale in market making are larger in thinner markets and for niche products. In Appendix B.5 we numerically show that these comparative statics also hold in an alternative model of niche products involving liner virtual types, where products become more niche as the elasticity of both demand and supply increase. Figure 4 highlights the importance of market thickness in markets for niche products, suggesting that it may be undesirable for a regulator to attempt to break up thick markets for such products. However, as we shall see in the next section, niche products may also increase the market power of monopoly market makers.

4.2 Competing bilateral exchanges mitigate market power for mass products

The double coincidence of wants problem is less severe for mass products than for niche products because the probability of finding a trading partner in, say, a bilateral trade setting is larger. This suggests that bilateral trade offers a viable outside option for mass products, and less so for niche products. Since outside options reduce buyers' willingness to pay and increase sellers' reservation prices, one would expect that in mass product markets, the market power of a thick market monopoly is constrained by the outside option offered by bilateral trade. We are now going to formalize this notion by extending the model to allow for the possibility that a perfectly thick large market coexists with bilateral trade.

Without this interpretation being necessary, one can think of the large market monopoly as a digital platform and of buyers and sellers as living in otherwise disconnected, geographical markets such as different towns or countries, each of which is connected to the platform. In line with the main analysis of this paper, we think of these local markets as being characterized by bilateral trade.

To fix ideas, we assume that trade in a bilateral exchange occurs via a posted price mechanism à la Hagerty and Rogerson (1987), where the posted price is the Walrasian price p. After learning its type each agent has the option of trading via the thick market monopoly or in a bilateral exchange where trade occurs if and only if the buyer and seller are willing to trade at p. For simplicity, assume that we have a regular mechanism design problem, which is to say that F and G exhibit increasing virtual type functions. This implies that if the outside option of agents is 0, posting prices of p_B and p_S is optimal for the monopoly. Even when agents have the outside option of joining a bilateral exchange, we assume that the monopoly is restricted to posting prices.[13]

While there may be multiple equilibria, we focus on the equilibrium with monotone sorting in which for any $p_B > p > p_S$, all buyers with $v \in (\overline{v}, 1]$ and all sellers with $c \in [0, \underline{c})$ trade via the monopoly and all the other types join the bilateral exchanges, where $\overline{v} > p_B$ and $p_S > \underline{c}$ and $1 - F(\overline{v}) = G(\underline{c})$. Under the stipulated assumptions, the respective expected utilities of a buyer of type v and seller of type c from the bilateral exchange is

$$U_B(v) := (1-\nu)(G(p) - G(\underline{c}))(v - p) \quad \text{and} \quad U_S(c) := (1-\nu)(F(\overline{v}) - F(p))(p - c),$$

where $v \geq p$ and $p \geq c$. If $v \geq p_B$ and $p_S \geq c$, the payoffs of these agents from going to the monopoly are $v - p_B$ and $p_S - c$, respectively. It follows that $\overline{v}$ and $\underline{c}$ satisfy $\overline{v} - p_B = U_B(\overline{v})$ and $p_S - \underline{c} = U_S(\underline{c})$.[14] Hence, for given $\overline{v}$ and $\underline{c}$, we have $p_B = \overline{v} - U_B(\overline{v})$ and $p_S = \underline{c} + U_S(\underline{c})$, which implies that the monopoly's spread is

$$p_B - p_S = \overline{v} - \underline{c} - (U_B(\overline{v}) + U_S(\underline{c})). \tag{4}$$

[13] If agents have the outside option of joining a bilateral exchange, the optimal mechanism for the monopoly is not known.

[14] Note that since $U_B'(v) < 1$ and $U_S'(c) > -1$ the single crossing condition holds.

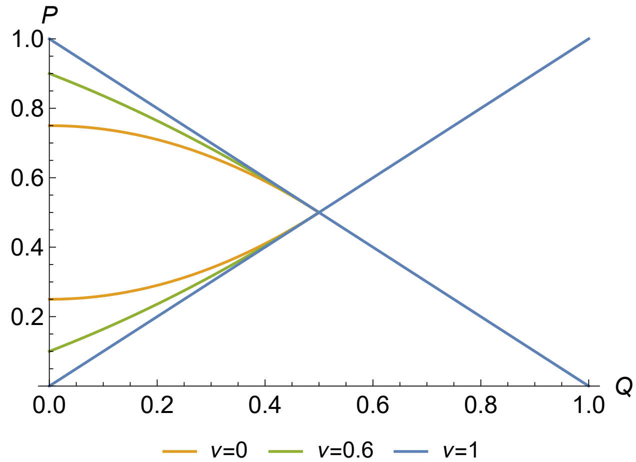


Figure 5: The inverse demand and supply functions faced by the monopoly when agents have bilateral trade as an outside option and F and G are uniform distributions.

Since it is never optimal to choose p_B and p_S such that $1 - F(\overline{v}) \neq G(\underline{c})$, we can replace $1-F(\overline{v})$ and $G(\underline{c})$ by the monopoly's "quantity" $q \in [0,1]$ (bearing in mind that if it chooses q its quantity traded will be $(1-\nu)q$). Replacing $F(p)$ by $1-w$ and $G(p)$ by w, the spread in (4) thus simplifies to

$$p_B - p_S = \left(F^{-1}(1-q) - G^{-1}(q)\right)(1-(1-\nu)(w-q)).$$

Figure 5 illustrates the effects of changes in ν on the inverse demand and supply functions the monopoly faces when the agents have the outside option of participating in a bilateral exchange, where the demand and supply functions are normalized by $1-\nu$. The monopoly's profit maximization problem can thus be written

$$\max_{q\in[0,1]} \left(F^{-1}(1-q) - G^{-1}(q)\right)(1-(1-\nu)(w-q))q, \tag{5}$$

which has the same logic and structure as that underlying (3). Denote the solution to this problem by $q^*(\nu)$.[15] Denoting the prices associated with $q^*(\nu)$ by $p_B^*(\nu)$ and $p_S^*(\nu)$, we have

$$p_B^*(\nu) = F^{-1}(1-q^*(\nu)) - (F^{-1}(1-q^*(\nu)) - p)(1-\nu)(w-q^*(\nu)) \tag{6}$$

and

$$p_S^*(\nu) = G^{-1}(q^*(\nu)) + (p - G^{-1}(q^*(\nu)))(1-\nu)(w-q^*(\nu)). \tag{7}$$

[15] We do not scale this profit by $1-\nu$ to make sure the maximization problem is well defined even as $\nu \to 1$ and to isolate the effects of ν via the outside options; the objective (5) can be thought of as a normalized profit.

Proposition 5. *Suppose that* $\pi(q) := (F^{-1}(1-q) - G^{-1}(q))\, q$ *is concave. Then* $q^*(\nu)$ *is strictly decreasing in* ν *with* $q^*(1) = m$ *and the spread* $p_B^*(\nu) - p_S^*(\nu)$ *is increasing in* ν.

Proposition 5 provides a concise formalization of the notion that mass products mitigate market power. As products become more niche and ν increases, the monopoly's quantity in the presence of competing bilateral exchanges decreases and its pricing becomes more aggressive. Both effects reduce the sum of consumer and producer surplus at the monopoly. If F and G are symmetric in such a way that $p_B^*(\nu) = 1 - p_S^*(\nu)$ (for example, if F and G correspond to uniform distributions), then the decreasing nature of the spread implies that $p_S^*(\nu)$ decreases in ν and $p_B^*(\nu)$ increases in ν. Thus, (normalized) consumer surplus and (normalized) producer surplus at the monopoly each decrease in ν.

The fact that $q^*(\nu) > m$ for any $\nu < 1$ is of independent interest. An increase in the monopoly's quantity decreases the probability of trade in the bilateral exchange and, consequently, the payoffs $U_B(\overline{v})$ and $U_S(\underline{c})$. Therefore, the increase in the quantity traded by the monopoly, relative to the model without bilateral exchanges, is driven by a foreclosure motive.

In Appendix B.6 we perform an analogous analysis, assuming that the second-best mechanism operates in the competing exchange and show that the key results derived here continue to hold.

A number of messages emerge from the analysis in this section. First, market power is naturally mitigated with mass products because bilateral trade offers viable outside options. Consequently, abuse of market power is less of a concern with thick market monopolies for mass products. By the same token, it is in the interest of the monopoly to drive out even small (bilateral) exchanges because these offer viable outside options and constrain its profit. Of course, in practice and in richer models, the monopoly may take additional or alternative actions to drive out small rivals. This suggests that for mass products, the main concern or focus should be on the foreclosure motive rather than on abuse of market power. Second, for niche products, small bilateral exchanges provide a less viable outside option for traders precisely because the monopoly generates more consumer and producer surplus than bilateral trade for sufficiently large ν. In other words, for niche products there is no natural mitigation of market power in the face of bilateral exchanges. Consequently, abuse of market power is a greater concern and foreclosure of small exchanges is of less concern for niche products.[16]

[16] As mentioned in the introduction, the competing bilateral exchanges we analyze differ from the competing exchanges the literature has mostly focused on to date; see, for example, Gehrig (1993), Spulber (2002), Rust and Hall (2003), and Loertscher and Niedermayer (2020). Even though matching in the competing exchange in a model such as Gehrig's is random, the competing exchange is a thick market because there is a continuum of agents, and these agents participate only if they expect positive surplus from so doing. Hence, even if

5 Conclusions

In this paper, we use an independent private values model to analyze the social costs and benefits of thick markets relative to thin markets. We distinguish between niche and mass products and model thin markets as bilateral exchanges and thick markets as markets with a continuum of buyers and sellers. We show that whether a product is niche or mass neither affects the severity of the incentive problem in thin markets nor the severity of the harm to consumers, producers and social surplus under a thick market monopoly relative to an efficient thick market. In contrast, the consumer, producer and social surplus benefits of thick markets relative to thin markets increase as the products become more niche and are unbounded as they become perfectly niche. Thus, for products that are sufficiently niche, the benefits of thick market monopolies are first order and the harm from monopolies relative to efficient thick markets are second order. These results are driven by the fact that for niche products thick markets overcome the double coincidence of wants problem that haunts thin markets. By the same token we show that competing bilateral exchanges constrain the market power of thick market monopolies for mass products but not for niche products. This provides some rationale for increasing regulatory scrutiny of thick market monopolies for niche products.

References

Anderson, C. (2006): *Long Tail: Why the Future of Business Is Selling Less of More*, New York: Hyperion.

Aumann, R. J. (1964): "Markets with a Continuum of Traders," *Econometrica*, 32, 39–50.

Brynjolfsson, E., Y. Hu, and M. D. Smith (2003): "Consumer Surplus in the Digital Economy: Estimating the Value of Increased Product Variety at Online Booksellers," *Management Science*, 49, 1580–1596.

Bulow, J. and P. Klemperer (1996): "Auctions versus Negotiations," *American Economic Review*, 86, 180–94.

Bulow, J. and J. Roberts (1989): "The Simple Economics of Optimal Auctions," *Journal of Political Economy*, 97, 1060–1090.

Cournot, A. (1838): *Recherceches sur les Principles Mathématiques de la Théorie des Richesses*, Paris.

Cripps, M. W. and J. M. Swinkels (2006): "Efficiency of Large Double Auctions," *Econometrica*, 74, 47–92.

Gehrig, T. (1993): "Intermediation in Search Markets," *Journal of Economics & Management Strategy*, 2, 97–120.

matching were random, such a thick competing exchange would eliminate the double coincidence of wants problem and would thus be a severe competitive constraint for a thick market monopoly for niche products.

Gresik, T. and M. Satterthwaite (1989): "The Rate at which a Simple Market Converges to Efficiency as the Number of Traders Increases: An Asymptotic Result for Optimal Trading Mechanisms," *Journal of Economic Theory*, 48, 304–332.

Hagerty, K. M. and W. P. Rogerson (1987): "Robust Trading Mechanisms," *Journal of Economic Theory*, 42, 94–107.

Kojima, F. and T. Yamashita (2017): "Double Auction with Interdependent Valuation: Incentives and Efficiency," *Theoretical Economics*, 1393–1438.

Loertscher, S. and C. Mezzetti (2021): "A Dominant Strategy, Double Clock Auction with Estimation-Based Tâtonnement," *Theoretical Economics*, 16, 943–978.

Loertscher, S. and E. V. Muir (2022): "Monopoly pricing, optimal randomization, and resale," *Journal of Political Economy*, 130, 566–635.

Loertscher, S., E. V. Muir, and P. G. Taylor (2022): "Optimal market thickness," *Journal of Economic Theory*, 200.

Loertscher, S. and A. Niedermayer (2020): "Entry-Deterring Agency," *Games and Economic Behavior*, 119, 172–188.

McAfee, R. P. (1992): "A Dominant Strategy Double Auction," *Journal of Economic Theory*, 56, 434–450.

Mussa, M. and S. Rosen (1978): "Monopoly and Product Quality," *Journal of Economic Theory*, 18, 301–317.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58–78.

Myerson, R. and M. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29, 265–281.

Rostek, M. and M. Weretka (2008): "Thin markets," in *The New Palgrave Dictionary of Economics, Online Edition.*

Rust, J. and G. Hall (2003): "Middlemen versus Market Makers: A theory of Competitive Exchange," *Journal of Political Economy*, 111, 353–403.

Rustichini, A., M. A. Satterthwaite, and S. R. Williams (1994): "Convergence to Efficiency in a Simple Market with Incomplete Information," *Econometrica*, 62, 1041–1063.

Satterthwaite, M. and S. R. Williams (1989): "Bilateral Trade with the Sealed k-Double Auction: Existence and Efficiency," *Journal of Economic Theory*, 48, 107–133.

——— (2002): "The Optimality of a Simple Market Mechanism," *Econometrica*, 70, 1841–1863.

Spulber, D. F. (2002): "Market Microstructure and the Incentives to Invest," *Journal of Political Economy*, 110, 352–381.

Tatur, T. (2005): "On the Tradeoff Between Deficit and Inefficiency and the Double-Auction with a Fixed Transation Fee," *Econometrica*, 73, 517–570.

Waldfogel, J. (2017): "How Digitization Has Created a Golden Age of Music, Movies, Books, and Television," *Journal of Economic Perspectives*, 31, 195–214.

Williamson, O. (1985): *The Economic Institutions of Capitalism*, New York: The Free Press.

Wilson, R. B. (1993): *Nonlinear Pricing*, New York: Oxford University Press.

Appendix

A Proofs

A.1 Proof of Lemma 1

Proof. The proof is confined to the weighted virtual value function as the proof for the weighted virtual cost function is analogous. Using $F(v) = \frac{F_\nu(v) - F_\nu(0)}{1 - F_\nu(0)}$ and $f(v) = \frac{f_\nu(v)}{1 - F_\nu(0)}$, we have $\Phi_\alpha(v) = v - \alpha \frac{1-F(v)}{f(v)} = v - \alpha \frac{\frac{1-F_\nu(v)}{1-F_\nu(0)}}{\frac{f_\nu(v)}{1-F_\nu(0)}} = v - \alpha \frac{1-F_\nu(v)}{f_\nu(v)}$ as claimed. □

A.2 Proof of Proposition 1

Proof. Recall that for any $\alpha \in [0, 1]$ we let $\overline{\Phi}_\alpha(v)$ and $\overline{\Gamma}_\alpha(c)$ denote the weighted *ironed* virtual valuation and cost functions, respectively. Due to the truncation invariance of these functions, they apply regardless of the value of the niche product parameter ν. The mechanism that maximizes the convex combination of welfare and profit with a weight of α on profit then has the allocation rule $Q^\alpha(v, c) = \mathbb{1}(\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c))$ that induces trade if and only if $\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c)$. Therefore, for any $\alpha \in [0, 1]$, the mechanism that implements the allocation rule Q^α is uniquely pinned down and does not vary with the parameter ν.[17] For any $\alpha \in [0, 1]$, we thus have

$$\begin{aligned}
S_1^\alpha(\nu) &= \mathbb{E}[(v - c)Q^\alpha(v, c)] \\
&= \mathbb{E}[(v - c)\mathbb{1}(\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c))] \\
&= \int_0^1 \int_0^1 (v - c)\mathbb{1}(\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c)) f_\nu(v) g_\nu(c) \, dc \, dv \\
&= (1 - \nu)^2 \int_0^1 \int_0^1 (v - c)\mathbb{1}(\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c)) f(v) g(c) \, dc \, dv \\
&= (1 - \nu)^2 \mathbb{E}[(v - c)Q^\alpha(v, c)] \\
&= (1 - \nu)^2 S_1^\alpha(0).
\end{aligned}$$

Using $S_1^{FB}(\nu) = (1 - \nu)^2 S_1^{FB}(0)$ then yields $\frac{S_1^\alpha(\nu)}{S_1^{FB}(\nu)} = \frac{S_1^\alpha(0)}{S_1^{FB}(0)}$, which shows that this ratio is independent of ν as required. □

[17] When $\alpha = 0$, we obtain a unique mechanism by taking the welfare-maximizing mechanism that otherwise maximizes the designer's profit.

A.3 Proof of Proposition 2

Proof. Using $S_\infty(\nu) = (1-\nu)S_\infty(0)$, $S_1^{FB}(\nu) = (1-\nu)^2 S_1^{FB}(0)$ and the definition of s_∞ yields $\frac{S_\infty(\nu)}{S_1^{FB}(\nu)} = \frac{s_\infty}{1-\nu}$. Moreover, since $\nu < 1$ and $s_\infty > 1$, we have $\frac{S_\infty(\nu)}{S_1^{FB}(\nu)} > 1$. Taking the limit as $\nu \to 1$ yields the final statement of the proposition. $\square$

A.4 Proof Proposition 3

Proof. We begin by proving the first statement of the proposition. For the sake of notational brevity, we write CS and PS instead of CS_∞^M and PS_∞^M throughout the proof of this first statement. We have already proven the proposition for the regular case. In general (that is, without imposing regularity) the optimal quantity of the large mass product monopoly, denoted m, is such that

$$\overline{\Phi}(F^{-1}(1-m)) = \overline{\Gamma}(G^{-1}(m)),$$

where $\overline{\Phi}(v) \equiv \overline{\Phi}_1(v)$ and $\overline{\Gamma}(c) \equiv \overline{\Gamma}_1(c)$.[18] It remains to determine the associated pricing mechanism. If $\overline{\Phi}(F^{-1}(1-m)) = \Phi(F^{-1}(1-m))$ and $\overline{\Gamma}(G^{-1}(m)) = \Gamma(G^{-1}(m))$, the problem is equivalent to the regular case analyzed in Section 3.3, and we are done. There are two additional generic and symmetric cases that need to be considered: (i) $\overline{\Phi}(F^{-1}(1-m)) > \Phi(F^{-1}(1-m))$ and $\overline{\Gamma}(G^{-1}(m)) = \Gamma(G^{-1}(m))$ and (ii) $\overline{\Phi}(F^{-1}(1-m)) = \Phi(F^{-1}(1-m))$ and $\overline{\Gamma}(G^{-1}(m)) < \Gamma(G^{-1}(m))$.[19]

In case (i), sellers are paid $p_S = G^{-1}(m)$ and producer surplus is $m(1-\nu)\mathbb{E}[p_S - c|c \leq p_S]$ (as it is in the regular case). Letting $[F^{-1}(1-m_2), F^{-1}(1-m_1)]$ denote the ironed range of $\overline{\Phi}$ containing $F^{-1}(1-m)$, buyers with values above $F^{-1}(1-m_1)$ are always served and pay $p_B^1 = \frac{m_2-m}{m_2-m_1}F^{-1}(1-m_1) + \frac{m-m_1}{m_2-m_1}F^{-1}(1-m_2)$, while buyers with values $v \in [F^{-1}(1-m_2), F^{-1}(1-m_1)]$ participate in a lottery where they are served with probability $\frac{m-m_1}{m_2-m_1}$ and pay $p_B^2 = F^{-1}(1-m_2)$ if they obtain a unit (see Loertscher and Muir, 2022). Consumer

[18] This is a straightforward generalization of the analysis in Section 3 of Loertscher and Muir (2022) to two-sided private information.

[19] At first glance it might appear as though we need to consider a third case where $\overline{\Phi}(F^{-1}(1-m)) > \Phi(F^{-1}(1-m))$ and $\overline{\Gamma}(G^{-1}(m)) < \Gamma(G^{-1}(m))$. This case is knife-edge because it requires that the constant regions of the ironed virtual type functions coincide. In this case there are necessarily multiple quantities that are profit-maximizing for the monopoly and at least one such quantity coincides with a previous case. That is, without loss of generality we can restrict attention to mechanisms that involve rationing on at most one side of the market. Within the set of profit-maximizing quantities, consumer and producer surplus are maximized at the largest quantity; see display (8) in Loertscher and Muir (2022) for consumer surplus (the results for producer surplus are analogous). Our results continue to hold for any monotone selection of the optimal quantity.

surplus with mass products is (see Loertscher and Muir, 2022, p.15)

$$\overline{CS}(0) = \int_0^{m_1} F^{-1}(1-x)dx + \frac{m-m_1}{m_2-m_1}\int_{m_1}^{m_2} F^{-1}(1-x)dx - \overline{R}(m),$$

where $\overline{R}(m) = \int_0^m \overline{\Phi}(F^{-1}(1-x))dx$.

In case (ii), buyers pay $p_B = F^{-1}(1-m)$ and consumer surplus is $m(1-\nu)\mathbb{E}[v - p_B | v \geq p_B]$ (as it is in the regular case). Letting $[G^{-1}(m_1), G^{-1}(m_2)]$ denote the ironed range of $\overline{\Gamma}$ containing $G^{-1}(m)$, sellers with costs below $G^{-1}(m_1)$ are always served and are paid $p_S^1 = \frac{m_2-m}{m_2-m_1}G^{-1}(m_1) + \frac{m-m_1}{m_2-m_1}G^{-1}(m_2)$, while sellers with costs $c \in [G^{-1}(m_1), G^{-1}(m_2)]$ participate in a lottery where they are given the chance to produce with probability $\frac{m-m_1}{m_2-m_1}$ and are paid $p_S^2 = G^{-1}(m_2)$ if they do so. Accordingly, producer surplus is

$$\overline{PS}(0) = \overline{C}(m) - \int_0^{m_1} G^{-1}(x)dx - \frac{m-m_1}{m_2-m_1}\int_{m_1}^{m_2} G^{-1}(x)dx,$$

where $\overline{C}(m) = \int_0^m \overline{\Gamma}(G^{-1}(x))dx$ is the convex hull of the cost of procuring m units.

Truncation invariance ensures that cases (i) and (ii) do not vary with $\nu \in [0,1)$. In particular, denote per buyer consumer surplus by $CS(\nu)$ and per seller producer surplus by $PS(\nu)$. Consumer and producer surplus in cases (i) and (ii) are then, respectively,

$$CS(\nu) = m(1-\nu)\mathbb{E}[v - p_B | v \geq p_B] \quad \text{and} \quad PS(\nu) = (1-\nu)\overline{PS}(0) \tag{8}$$

and

$$CS(\nu) = (1-\nu)\overline{CS}(0) \quad \text{and} \quad PS(\nu) = m(1-\nu)\mathbb{E}[p_S - c | c \leq p_S]. \tag{9}$$

Combining all of these cases we have

$$\frac{CS(\nu)}{S_1^{FB}(\nu)} = \frac{1}{1-\nu}cs \quad \text{and} \quad \frac{PS(\nu)}{S_1^{FB}(\nu)} = \frac{1}{1-\nu}ps \tag{10}$$

where

$$cs := \frac{CS(0)}{S_1^{FB}(0)} \quad \text{and} \quad ps := \frac{PS(0)}{S_1^{FB}(0)}.$$

Observe that $cs > 0$ and $ps > 0$. Thus, the ratios are increasing in ν. Letting

$$\hat{\nu} = \max\{1 - cs, 1 - ps\} \tag{11}$$

the first statement of the proposition follows.

We now prove the second statement of the proposition. Taking the limit as $\nu \to 1$ in (10) yields $\lim_{\nu\to 1} \frac{CS_\infty^M(\nu)}{S_1^{FB}(\nu)} = \lim_{\nu\to 1} \frac{PS_\infty^M(\nu)}{S_1^{FB}(\nu)} = \infty$ as required. From (8) and (9) we also see that $CS_\infty^M(\nu)$ and $PS_\infty^M(\nu)$ are directly proportional to $1-\nu$. Moreover, our previous analysis implies that the optimal mechanism of the monopoly does not vary with ν and hence monopoly profits are also directly proportional to $1-\nu$. Combining this with the expressions for $CS_\infty(\nu)$, $PS_\infty(\nu)$ and $S_\infty(\nu)$ provided in Section 3.3 shows that the ratios $\frac{CS_\infty^M(\nu)}{CS_\infty(\nu)}$, $\frac{PS_\infty^M(\nu)}{PS_\infty(\nu)}$ and $\frac{S_\infty^M(\nu)}{S_\infty(\nu)}$ are finite and independent of ν for $\nu \in [0,1)$, which concludes the proof. $\square$

A.5 Proof of Proposition 4

Proof. Suppose that we have a market with n buyers with valuations $v_1,\dots,v_n$ and m sellers with costs $c_1,\dots,c_m$. Observing that an ex post efficient allocation gives the m objects to the m agents with the largest types in the set $X = \{v_1,\dots,v_n,c_1,\dots,c_m\}$, it follows that a price is market-clearing if and only if it is between the mth and $m+1$st highest elements in X, which is equivalent being between the nth lowest and $n+1$st lowest elements of X. Thus, the lowest Walrasian price given X is $\underline{p}(X) := x_{(n)}$ and the highest Walrasian price is $\overline{p}(X) := x_{(n+1)}$. Notice that $\overline{p}(X)$ is either a buyer's value or a seller's cost, and likewise for $\underline{p}(X)$. We then have the following well-known observation that $\overline{p}(X)$ is the social marginal product of a seller with cost $c < \overline{p}(X)$ who arrives to a market characterized by X, since either the arriving seller replaces another seller with a higher cost (which happens if $\overline{p}(X)$ is a cost) or it expands the quantity traded by one unit (which happens if $\overline{p}(X)$ is a value). If the arriving seller's cost is above $\overline{p}(X)$, its social marginal product is 0. Likewise, $\underline{p}(X)$ is the social marginal cost of a buyer with value $v > \underline{p}(X)$ who arrives to a market characterized by X, since either the arriving buyer replaces another buyer with a lower value (which happens if $\underline{p}(X)$ is a value) or it expands the quantity traded by one unit (which happens if $\underline{p}(X)$ is a cost). If the arriving buyer's value is below $\underline{p}(X)$, its social marginal product is 0. Thus, with an additional buyer with value v social surplus increases by $\max\{v - \underline{p}(X), 0\}$ and with an additional seller with cost c, social surplus increases by $\max\{\overline{p}(X) - c, 0\}$. These last observations follow from the fact that with single-unit demands and supplies, $\underline{p}(X)$ are the trading buyers' and $\overline{p}(X)$ are the trading sellers' VCG payments, which represent the social marginal cost and products of these agents.

If an additional seller with cost c is added to a market with n buyers and n sellers, its expected welfare contribution is thus

$$\mathbb{E}[(x_{(n)} - c)\mathbb{1}(c \le x_{(n)})] = \int_0^1 \int_0^x (x-c) g_\nu(c) h_{\nu,n,n,n}(x)\, dc dx.$$

Similarly, if an additional buyer with valuation v is then added to a market with n buyers and $n+1$ sellers, its expected welfare contribution is thus

$$\mathbb{E}[(v - x_{(n+1)})\mathbb{1}(v \geq x_{(n+1)})] = \int_0^1 \int_x^1 (v-x) f_\nu(v) h_{\nu,n+1,n,n+1}(x)\, dvdx.$$

Putting all of this together yields the statement in the proposition. □

A.6 Proof of Proposition 5

Proof. We start by noting that the profit function in (5) can be rewritten as

$$\Pi(q) := \pi(q)(1 - (1-\nu)(w-q)).$$

Note that for $\nu \in [0,1)$ we must have $q^*(\nu) \in [0,w)$ by the definition of w, the monotonocity of the supply and demand functions and the fact that the monopoly always optimally charges a positive bid-ask spread. This also implies that for any $q \in [0,w)$, we have $\pi(q) > 0$.

Differentiating the profit function with respect to q yields

$$\Pi'(q) = \pi'(q)(1-(1-\nu)(w-q)) + \pi(q)(1-\nu).$$

By construction, $\pi'(m) = 0$ and $\pi(m) > 0$. Thus, for $\nu \in [0,1)$, we have $\Pi'(m) > 0$. Suppose that $\nu \in [0,1)$ and that $q^* < w$ satisfies the first-order condition $\Pi'(q^*) = 0$. This implies that $(1-\nu)\pi(q^*) > 0$ and $1-(1-\nu)(w-q) > 0$ and, consequently, $\pi'(q^*) < 0$. Taking the second derivative of the profit function with respect to q, we have

$$\Pi''(q) = \pi''(q)(1-(1-\nu)(w-q)) + 2\pi'(q)(1-\nu).$$

Then if $\nu \in [0,1)$ and q^* satisfies $\Pi'(q) = 0$, we have $\pi'(q^*) < 0$ (by our previous observation) and $\pi''(q^*) \leq 0$ (since, by assumption, π is a concave function). Combining these facts shows that if $\Pi'(q) = 0$ holds, then we necessarily have $\Pi''(q) < 0$. Thus, Π is a quasiconcave function and the second-order condition for a maximum is satisfied whenever the first-order condition holds.

Now that we have established the first-order condition that characterizes q^*, we determine how q^* varies with ν. Differentiating the first-order condition

$$\Pi'(q) = \pi'(q^*(\nu))(1-(1-\nu)(w-q^*(\nu))) + (1-\nu)\pi(q^*(\nu)) = 0$$

with respect to ν and dropping the argument of $q^*(\nu)$ for the sake of brevity yields

$$\pi'(q^*)\left(w - q^* + (1-\nu)\frac{dq^*}{d\nu}\right) + \pi''(q)(1-(1-\nu)(w-q^*))\frac{dq^*}{d\nu} = 0$$

Rearranging this expression, we have

$$\left[\pi''(q)(1-(1-\nu)(w-q^*)) + 2(1-\nu)\pi'(q^*)\right]\frac{dq^*}{d\nu} = -\left(\pi'(q^*)(w-q^*) - \pi(q^*)\right),$$

and hence

$$\frac{dq^*}{d\nu} = -\frac{\pi'(q^*)(w-q^*) - \pi(q^*)}{\pi''(q^*)(1-(1-\nu)(w-q^*)) + 2(1-\nu)\pi'(q^*)}.$$

Suppose $\nu \in [0,1)$. Then since π is concave (which implies that $\pi''(q^*) \le 0$), $\pi'(q^*) < 0$ and $q^* < w$ we have $\pi''(q^*)(1-(1-\nu)(w-q^*)) + 2(1-\nu)\pi'(q^*) < 0$. Moreover, since $\pi'(q^*) < 0$, we also have $\pi'(q^*)(w-q^*) - \pi(q^*) < 0$. Putting all of this together yields $\frac{dq^*}{d\nu} < 0$ and $q^*(\nu)$ is strictly decreasing in ν as required.

We are left to establish that the spread $p_B^*(\nu) - p_S^*(\nu)$ is increasing in ν. To see this, let

$$\pi^{BT}(q,\nu) := \left(F^{-1}(1-q) - G^{-1}(q)\right)(1-(1-\nu)(w-q))q$$

be the profit function in the presence of bilateral exchanges. A revealed preference argument shows that $\pi^{BT}(q^*(\nu),\nu)$ is increasing in ν. To see this, consider $\nu_0 < \nu_1$. Then because the monopoly could keep the quantity at $q^*(\nu_0)$ when the parameter is $\nu_1 > \nu_0$, we have

$$\begin{aligned}
\pi^{BT}(q^*(\nu_1),\nu_1) &> \pi^{BT}(q^*(\nu_0),\nu_1) \\
&= \left(F^{-1}(1-q^*(\nu_0)) - G^{-1}(q^*(\nu_0))\right)(1-(1-\nu_1)(w-q^*(\nu_0)))q^*(\nu_0) \\
&> \left(F^{-1}(1-q^*(\nu_0)) - G^{-1}(q^*(\nu_0))\right)(1-(1-\nu_0)(w-q^*(\nu_0)))q^*(\nu_0) \\
&= \pi^{BT}(q^*(\nu_0),\nu_0).
\end{aligned}$$

Hence, $\pi^{BT}(q^*(\nu),\nu)$ is increasing in ν. By definition,

$$\pi^{BT}(q^*(\nu),\nu) = (p_B^*(\nu) - p_S^*(\nu))q^*(\nu).$$

Since $q^*(\nu)$ decreases in ν, $p_B^*(\nu) - p_S^*(\nu)$ must increase in ν for $\pi^{BT}(q^*(\nu),\nu)$ to be increasing in ν. $\square$

B Extensions and robustness

The setup presented in Section 2 was constructed so that the virtual valuation and cost functions were invariant to the niche product parameter ν. This feature together with the symmetry of the setup simplified the technical aspects of the analysis whilst providing a setting suitable for elucidating the key messages of this paper. However, the main insights and results of this paper extend beyond this setting as we now show.

B.1 Asymmetric masses of buyers and sellers

The first assumption that we can easily relax is the assumption that there is an equal mass of buyers and sellers on each side of the market in the thick Walrasian market setting analyzed in Section 3.2. In particular, without loss of generality, we can always normalize the mass of agents on one side of the market, say buyers, to 1. Let $\mu > 0$ denote the mass of sellers. Fixing $\mu > 0$, we then immediately recover the results of Proposition 1, Proposition 2 and Proposition 3. Graphically, we accommodate this extension by appropriately rescaling the inverse supply curves illustrated in Figure 2. That is, for each value of $\nu \in [0, 1)$ we replace the inverse supply curve $P^S\left(\frac{\cdot}{1-\nu}\right)$ with the rescaled inverse supply curve $P^S\left(\frac{\cdot}{\mu(1-\nu)}\right)$. This leaves us with a Walrasian price $p(\mu)$ and quantity $w(\mu)$ that do not vary with ν and, consequently, all of our results carry over to this setting.

B.2 Asymmetric niche product parameters

The next assumption that we can readily relax is the assumption that a symmetric niche product parameter ν applies to both sides of the market. Specifically, we can allow for a niche product parameter of ν_B that applies to the buyers' side of the market and a parameter of ν_S that applies to the sellers' side of the market. In this asymmetric setting, the Walrasian quantity traded and price now vary with both ν_B and ν_S. However, the virtual type functions are unaffected. We recover results analogous to those stated in Proposition 1, Proposition 2 and Proposition 3 if we take the limit as $\nu_B \to 1$ and $\nu_S \to 1$ in a manner such that the ratio $(1 - \nu_B)/(1 - \nu_S)$ is fixed.

B.3 Varying overlapping support of distributions

Our setup in Section 2 was constructed so that for any value of $\nu \in [0, 1)$, the region of overlapping support for the distributions F_ν and G_ν was always the interval $[0, 1]$. In this section we show that the spirit of our analysis in Section 3 extends to an alternative setting where the region of overlapping support varies with ν.

We again let ν denote the niche market parameter and construct the setup so that a product becomes increasingly niche as ν increases. However, in this alternative setup we take $\nu \in (-1, 0]$. When $\nu = 0$ we now assume that buyer values $v \in [0, 1]$ are drawn from a distribution F with density f that has full support on $[0, 1]$ and seller costs $c \in [0, 1]$ are drawn from a distribution G with density g that has full support on $[0, 1]$. In the market for a niche product with parameter $\nu \in (-1, 0)$ we assume that buyer values $v \in [a_\nu, 1]$ with $a_\nu > 0$ are drawn from a distribution F_ν with density f_ν that has full support on $[a_\nu, 1]$ and seller costs $c \in [0, b_\nu]$ with $b_\nu < 1$ are drawn from a distribution G_ν with density g_ν that has full support on $[0, b_\nu]$. We further assume that, for all $\nu \in (-1, 0)$, a_ν and b_ν are such that $F(a_\nu) = -\nu = 1 - G(b_\nu)$ and that, for all $v \in [a_\nu, 1]$ and $c \in [0, b_\nu]$,

$$F_\nu(v) = \frac{F(v) + \nu}{1 + \nu} \quad \text{and} \quad G_\nu(c) = \frac{G(c)}{1 + \nu}.$$

Notice that this implies that a_ν is decreasing in ν and b_ν is increasing in ν. We let $\underline{\nu}$ denote the largest value of $\nu \in (-1, 0]$ such that $a_\nu \geq b_\nu$.

In this setup, in a market for a product with niche parameter $\nu \in (-1, 0)$, buyers values are truncated from below so that buyer values are distributed on the interval $[a_\nu, 1]$ and seller costs are distributed on the interval $[0, b_\nu]$, where a_ν is decreasing in ν and b_ν is increasing in ν. This implies that increasing ν results in less favourable distributions in the sense that, for all $\nu, \nu' \in (-1, 1)$ with $\nu' > \nu$, $v \in [0, 1]$ and $c \in [0, 1]$, $F_{\nu'}(v) \geq F_\nu(v)$ and $G_\nu(c) \geq G_{\nu'}(c)$. We let Φ and Γ respectively denote the virtual valuation and virtual cost function that correspond to the distributions F and G. The truncation invariance of these functions imply that the same virtual valuation and virtual cost functions are valid for any $\nu \in (-1, 0)$, on the appropriately restricted domain. Observe that for $\nu \leq \underline{\nu}$, the two supports do not overlap. This implies that full trade is efficient and the first-best outcome is possible in the bilateral trade problem. Consequently, for $\nu \leq \underline{\nu}$, the Walrasian quantity traded is 1 and

$$\frac{S_1^{SB}(\nu)}{S_1^{FB}(\nu)} = \frac{S_\infty(\nu)}{S_1^{FB}(\nu)} = 1.$$

Thus, there are no benefits from thicker markets.

For all $\nu > \underline{\nu}$ we have $a_\nu < b_\nu$. Since the distribution supports overlap in this case, full trade is not efficient and the impossibility result of Myerson and Satterthwaite (1983) holds. Recall that for a given $\alpha \in [0, 1]$, we let $\overline{\Phi}_\alpha(v)$ and $\overline{\Gamma}_\alpha(c)$ denote, respectively, the weighted ironed virtual valuation and cost functions and $Q^\alpha(v, c) = \mathbb{1}(\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c))$ denote the allocation rule that induces trade if and only if $\overline{\Phi}_\alpha(v) \geq \overline{\Gamma}_\alpha(c)$. Recall also that the second-best mechanism maximizes equally weighted ex ante expected social surplus subject to incentive compatibility, individual rationality and no-deficit constraints. This mechanism is therefore characterized by the unique value of α such that the allocation rule $Q^{\alpha}(v, c)$ generates zero budget surplus. That is, the second-best mechanism is characterized by the unique value α_ν^* satisfying

$$\mathbb{E}[(\Phi(v) - \Gamma(c))Q^{\alpha}(v, c)] = 0.$$

In the setup from Section 2, the second-best mechanism (and hence α_ν^*) was invariant to ν. Here, as ν increases in value beyond $\underline{\nu}$, the Walrasian quantity traded $w(\nu)$ will decrease in ν and α_ν^* will become strictly positive and monotonically increase in ν. Formally, we have the following result.

Proposition B.1. *The parameter α_ν^* that characterizes the second-best mechanism increases in $\nu \in (-1, 0]$ with $\alpha_\nu^* = 0$ for all $\nu \leq \underline{\nu}$. Moreover, there exists $\overline{\nu} < 0$ such that $\alpha_\nu^* = \alpha_0^*$ for all $\nu \geq \overline{\nu}$.*

Proof. Let $\nu, \nu' \in (-1, 0]$ with $\nu' > \nu$. Let $R_1^{\alpha}(\nu)$ denote revenue under the allocation rule Q^{α}. Then by construction we have

$$R_1^{\alpha_\nu^*}(\nu) = 0$$

and

$$R_1^{\alpha_\nu^*}(\nu') = \int_0^1 \int_0^1 (\Phi(v) - \Gamma(c)) \mathbb{1}(\Phi_{\alpha_\nu^*}(v) \geq \Gamma_{\alpha_\nu^*}(c)) \, dF_{\nu'}(v) \, dG_{\nu'}(c).$$

Since the integrand $(\Phi(v) - \Gamma(c)) \mathbb{1}(\Phi_{\alpha_\nu^*}(v) \geq \Gamma_{\alpha_\nu^*}(c))$ is increasing in v and decreasing in c, the distribution F_ν first-order stochastic dominates the distribution $F_{\nu'}$ and the distribution $G_{\nu'}$ first-order stochastic dominates the distribution G_ν, we have

$$R_1^{\alpha_\nu^*}(\nu') \leq R_1^{\alpha_\nu^*}(\nu) = 0.$$

Hence, $\alpha_{\nu'}^* \geq \alpha_\nu^*$ as required. Since full trade is optimal for any $\nu \leq \underline{\nu}$, we have $\alpha_\nu^* = 0$ for all $\nu \leq \underline{\nu}$. Moreover, by the impossibility result of Myerson and Satterthwaite (1983), $\alpha_0^* > 0$. This implies that when $\nu = 0$, there exists cutoff types $\underline{v}^{SB} > 0$ and $\overline{c}^{SB} < 1$ such that any buyer with $v \leq \underline{v}^{SB}$ and any seller with $c \geq \overline{c}^{SB}$ never trade under the second-best mechanism. Thus, setting $\overline{v} = \min\{\nu \in (-1, 0) : a_\nu \leq \underline{v}^{SB}, b_\nu \geq \overline{c}^{SB}\}$ we have $\alpha_\nu^* = \alpha_0^*$ for all $\nu \geq \overline{\nu}$ as required. □

Proposition B.1 implies that $\frac{S_1^{SB}(\nu)}{S_1^{FB}(\nu)}$ decreases in ν. Since $w(\nu)$ is less than 1 and decreasing ν, it also follows that $\frac{S_\infty(\nu)}{S_1^{FB}(\nu)}$ increases in ν. Interestingly, α_ν^* will cease increasing in ν before

ν reaches 0, that is, before $a_\nu = 0$ and $b_\nu = 1$. Indeed, letting α^* denote the second-best mechanism parameter that corresponds to the case $\nu = 0$, we have

$$\alpha^*_\nu = \alpha^*$$

for all $\nu \geq \max\{\nu_0, \nu_1\}$, where ν_0 and ν_1 are such that $\Phi_{\alpha^*}(a_{\nu_0}) = 0$ and $\Gamma_{\alpha^*}(b_{\nu_1}) = 1$. For example, when F and G are uniform, we have $\nu_0 = \nu_1 = 1/4$ with $a_{\nu_0} = 1/4$ and $b_{\nu_1} = 3/4$.

To summarize, this extension merely increases the complexity of the model without altering the main messages that emerge from the analysis in Section 3.

B.4 Exponential distributions

An alternative way we can model niche products is to consider distributions with unbounded support. We can then create increasingly niche products by shifting probability mass towards the tails of the distributions in an appropriate fashion. For example, suppose we take $v \in [0, \infty)$ with $F(v) = 1 - e^{-v}$ and $c \in [\log(a), \infty)$ with $G(c) = 1 - ae^{-c}$, where $a > 0$ is a constant. This specification implies that in a perfectly thick market with a contiuum of traders the Walrasian price and quantity traded are given by $p(a) = \ln(1 + a)$ and $w(a) = 1/(1+a)$, respectively. Increasing the parameter a pushes more probability mass towards the right tail of the sellers' cost distribution and leads to a decrease in the Walrasian quantity. Thus, the parameter a plays a similar role to the parameter ν in the model from Section 2. Expressed in terms of a, we have

$$S_1^{FB}(a) = \begin{cases} \frac{1}{2}(a - 2\ln(a)), & 0 < a \leq 1 \\ \frac{1}{2a}, & a > 1 \end{cases}$$

and

$$S_\infty(a) = \ln(1 + a) - \ln(a).$$

This implies

$$\frac{S_\infty(a)}{S_1^{FB}(a)} = \begin{cases} \frac{2(\ln(1+a) - \ln(a))}{(a - 2\ln(a))}, & 0 < a \leq 1 \\ 2a(\ln(1 + a) - \ln(a)), & a > 1. \end{cases}$$

Since $a = 0$ corresponds to having $w = 1$ we have

$$\lim_{a \to 0} \frac{S_\infty(a)}{S_1^{FB}(a)} = 1.$$

Moreover, we also have

$$\lim_{a\to\infty} \frac{S_\infty(a)}{S_1^{FB}(a)} = 2.$$

Solving for the second-best mechanism, in this setting we have

$$\Phi_\alpha(v) = v - \alpha \quad \text{and} \quad \Gamma_\alpha(c,a) = \alpha\left(\frac{e^c}{a} - 1\right) + c.$$

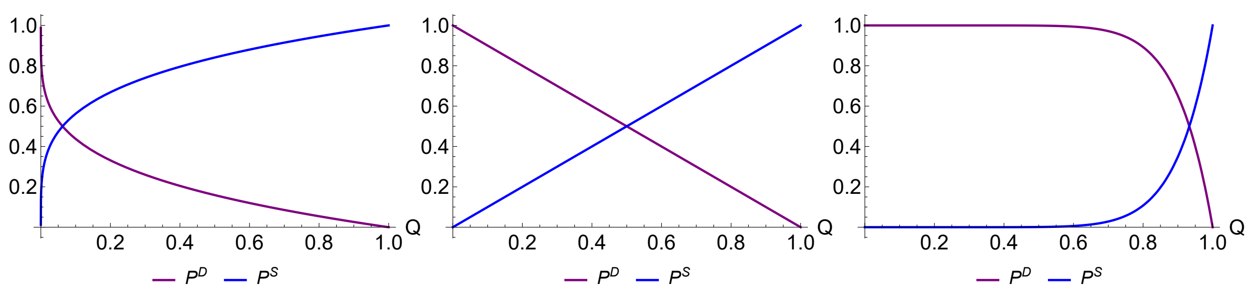
The value α_a^* that characterizes the second-best mechanism (see the previous subsection for a more detailed explanation of this) then satisfies

$$\frac{2}{3}e^{-\alpha}\left(\alpha + e^{\alpha}(\alpha-2)\alpha(\mathrm{Chi}(\alpha) - \mathrm{Shi}(\alpha)) - 1\right) = 0,$$

where the functions $\mathrm{Chi}(\cdot)$ and $\mathrm{Shi}(\cdot)$ denote the hyperbolic cosine integral and the hyperbolic sine integral, respectively. Numerical calculations then show that the ratio $\frac{S_1^{SB}(a)}{S_1^{FB}(a)}$ is decreasing in a and is bounded from below by 0.854367.

B.5 Linear virtual types

An alternative modelling approach is to consider settings in which products become more niche as the elasticity of supply and demand increases. To fix ideas we consider a setting with linear virtual type functions in which a single parameter $a > 0$ controls the elasticity of both demand and supply, as well as the Walrasian quantity. Specifically, we assume that buyers draw their values $v \in [0,1]$ from the distribution $F_a(v) = 1-(1-v)^a$ and sellers draw their costs $c \in [0,1]$ from the distribution $G_a(c) = c^a$. This yields a Walrasian quantity of $w(a) = 2^{-a}$ and price of $p = 1/2$.



(a) $a = 4$ (b) uniform ($a = 1$) (c) $a = 0.1$

Figure 6: Demand and supply for the specification with linear virtual types.

Thus, this model exhibits similar behavior with respect to the parameter a as the model in the main body does with respect to the parameter ν. Here, as a increases and a product becomes more niche, the demand and supply curves become increasingly elastic and the Walrasian quantity $w(a)$ decreases. Per trader welfare in the thick market limit is $S_\infty(a) = \int_0^{w(a)}(F^{-1}(1-q)-G^{-1}(q))dq$. Figure 6 displays demand and supply for three different values of a. As a becomes small (large), the Walrasian quantity goes to 1(0).

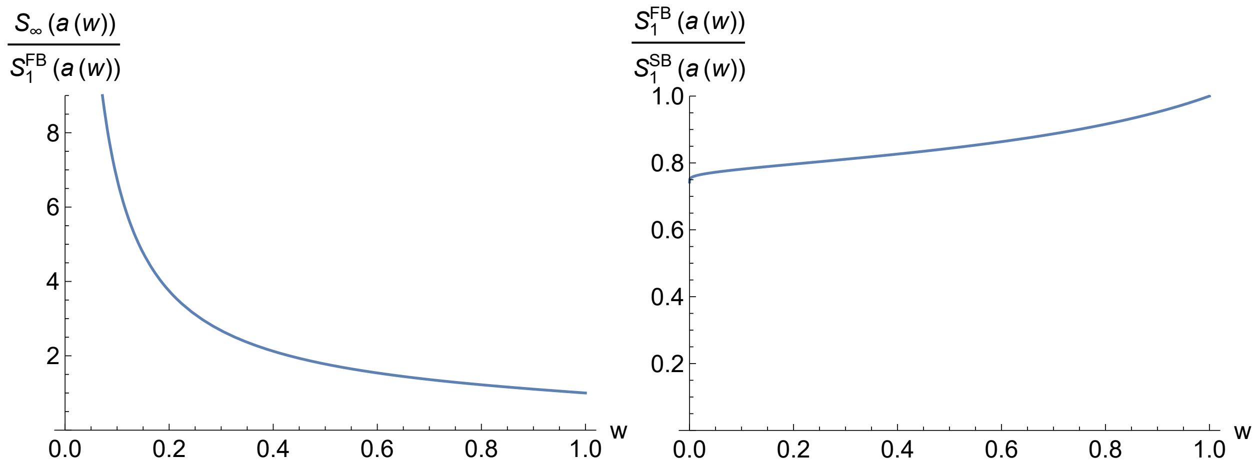


Figure 7: The matching benefits $S_\infty(a)/S_1^{FB}(a)$ of thick markets are unbounded as the Walrasian quantity goes to 0 (panel a). In contrast, the incentive benefits $S_1^{FB}(a)/S_1^{SB}(a)$ are bounded (panel b). In both panels we use $a = -\ln(w)/\ln(2)$.

In this setup, the weighted virtual type functions are given by

$$\Phi_{a,\alpha}(v) = v - \alpha\frac{1-v}{a} \quad \text{and} \quad \Gamma_{a,\alpha}(c) = c + \alpha\frac{c}{a}.$$

Although the weighted virtual type functions and hence the ratio $\frac{S_1^{SB}(a)}{S_1^{FB}(a)}$ now vary with the parameter a, we can still bound the relative cost of incentives.

Proposition B.2. *The ratio $S_1^{SB}(a)/S_1^{FB}(a)$ is decreasing in a for $a \geq 0$ and is bounded below by $2/e \approx 0.73$. In contrast, $\lim_{a\to\infty}\frac{S_\infty(a)}{S_1^{FB}(a)} = \infty$.*

Proof. Routine calculations yield

$$\frac{S_1^{SB}(a)}{S_1^{FB}(a)} = \frac{4^{-a}\left(\frac{a+1}{2a+1}\right)^{-2a}\Gamma(2a+2)}{(a+1)\Gamma(2a+1)} \quad \text{and} \quad \frac{S_\infty(a)}{S_1^{FB}(a)} = \frac{2^{-a}\Gamma(2+2a)}{a\Gamma(a)\Gamma(2+a)},$$

where (in a slight abuse of notation) here Γ denotes the gamma function. This is decreasing in a for $a \geq 0$ and taking the limit as $a \to \infty$ yields $2/e$. $\square$

With regard to the thick Walrasian market, Proposition 2 still holds as stated for this setup if we replace the limit $\nu \to 1$ with the limit $a \to \infty$. While the invariance result that

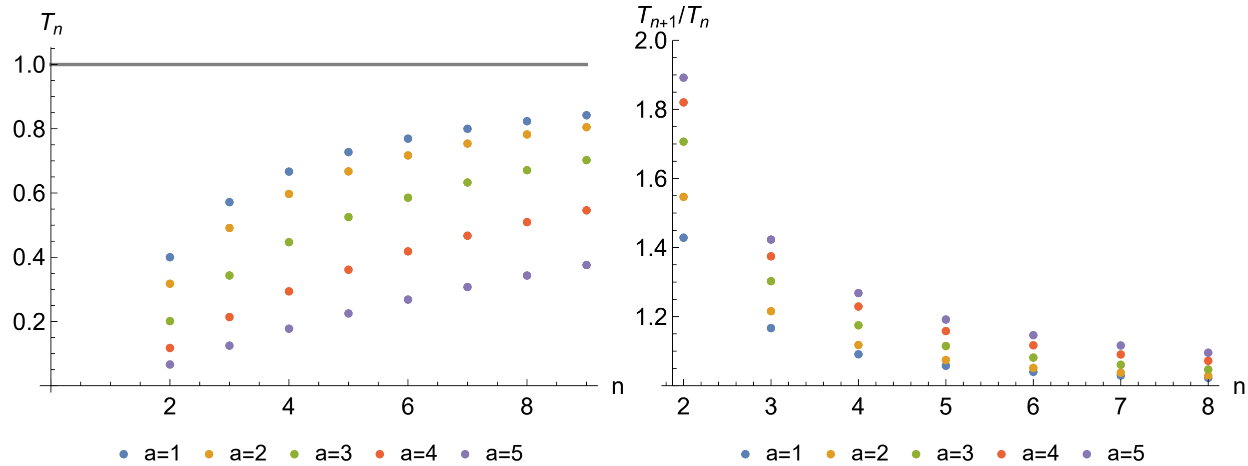


Figure 8: Illustration of our measure of market thickness T_n and returns to scale in market making T_{n+1}/T_n for uniform distributions and various values of a and n.

holds for the parameterization with truncation of Proposition 1 does not extend to the setting with linear virtual types, it remains to be the case that the variation in $S_1^{SB}(a)/S_1^{FB}(a)$ as one varies a is bounded—it goes from 0.73 as $a \to \infty$ to 1 as $a \to 0$—and dwarfed by the variation in S_∞/S_1^{FB}.

Consider now the thick market monopoly. Since the virtual type functions are linear (and hence monotone), the optimal trading mechanism consists of a posted price $p_B(a)$ for buyers and $p_S(a)$ for sellers. Since both the demand and supply schedules become increasingly elastic as a increases, the spread $p_B(a) - p_S(a)$ decreases in a and $\lim_{a\to\infty} p_B(a) = \lim_{a\to\infty} p_S(a) = p(a) = 1/2$. Proposition 3 also continues to hold as stated in this setting if we again replace the limit $\nu \to 1$ with $a \to \infty$.

With regard to the returns to scale in market making, we cannot analytically compute T_n and T_{n+1}/T_n in this model. However, the numerical calculations displayed in Figure 8 show that this model exhibits the same comparative statics as those in our baseline model.

B.6 Competing exchanges: Second-best bilateral trade

In this section we reconsider the analysis from Section 4.2 assuming that the second-best mechanism is used in the competing bilateral exchanges. For this analysis, we restrict ourselves to the case in which F and G are uniform.

As before, we focus on the equilibrium with monotone sorting in which all buyers with $v \in (\overline{v}, 1]$ and all sellers with $c \in [0, \underline{c})$ trade via the monopoly and all the other types join the bilateral exchanges. We let $U_B(v)$ denote the expected utility of a buyer of type v from the bilateral exchange and $U_S(c)$ denote that of a seller of type c. Routine calculations show that the second-best mechanism has $\alpha^* = 1/3$ and is such that

$$U_B(v) = \frac{1-\nu}{32}\left(\overline{v}^2 - 2\overline{v}(\underline{c} + 4v) - 15\underline{c}^2 + 8\underline{c}v + 16v^2\right)$$

and

$$U_S(c) = \frac{1-\nu}{32}\left(16c^2 + 8c(\overline{v} - \underline{c} - 4) - 15\overline{v}^2 - 2\overline{v}(\underline{c} - 12) + \underline{c}(\underline{c} + 8)\right).$$

It again follows that $\overline{v}$ and $\underline{c}$ satisfy[20]

$$\overline{v} - p_B = U_B(\overline{v}) \quad \text{and} \quad p_S - \underline{c} = U_S(\underline{c})$$

and the monopoly's spread is

$$p_B - p_S = \overline{v} - \underline{c} - (U_B(\overline{v}) + U_S(\underline{c})). \tag{12}$$

Again, noting that it is never optimal to choose p_B and p_S such that $1 - F(\overline{v}) \neq G(\underline{c})$, we can replace $1 - F(\overline{v})$ and $G(\underline{c})$ by the monopoly's "quantity" $q \in [0, 1]$. The monopolist's profit can then be written

$$\pi(q) = (1 - 2q - (U_B(\overline{v}) + U_S(\underline{c}))q = \frac{q}{16}(q - 2q)(9\nu - 6(1-\nu)q + 7).$$

The corresponding first-order condition that pins down the optimal quantity q^* is then given by

$$7 + 36(q^*)^2(1-\nu) + 9\nu - 8q^*(5 + 3\nu) = 0.$$

Solving this quadratic equation yields the two solutions

$$q^*(\nu) = \frac{10 + 6\nu + \sqrt{37 + 102\nu + 117\nu^2}}{18(1-\nu)}, \quad \frac{10 + 6\nu - \sqrt{37 + 102\nu + 117\nu^2}}{18(1-\nu)}.$$

Imposing the requirement that $q^*(\nu) \in [0, 1]$ then leaves us with

$$q^*(\nu) = \frac{10 + 6\nu - \sqrt{37 + 102\nu + 117\nu^2}}{18(1-\nu)}.$$

Note that in this case $q^*(\nu)$ is increasing in ν. The spread of the monopolist is then given

[20]Note that since $U_B'(v) < 1$ and $U_S'(c) > -1$ the single crossing condition holds.

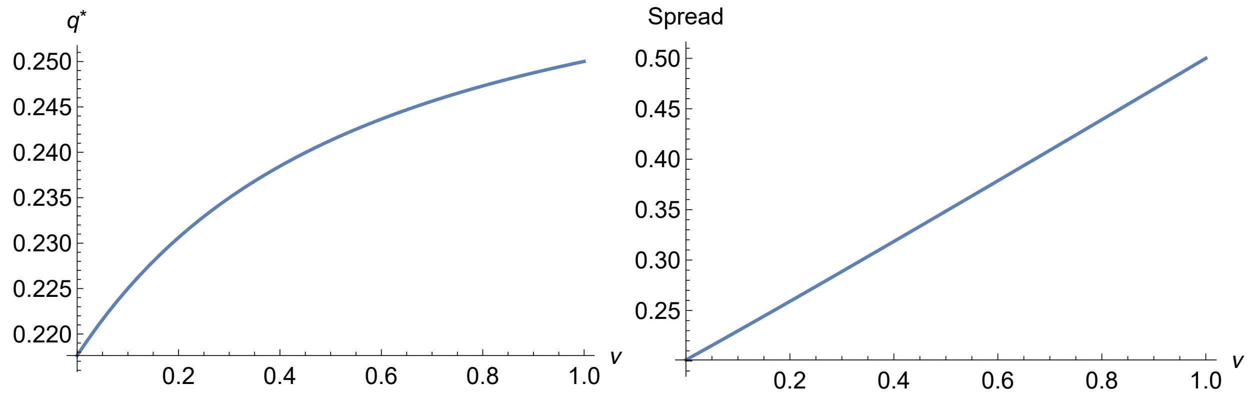


Figure 9: Equilibrium quantity $q^*(\nu)$ (panel (a)) and the equilibrium spread $p_B^*(\nu) - p_S^*(\nu)$ (panel (b)) as a function of ν for F and G uniform.

by

$$p_B - p_S = \frac{-99\nu^2 + 3\left(\sqrt{117\nu^2 + 102\nu + 37} - 14\right)\nu + 5\sqrt{117\nu^2 + 102\nu + 37} + 13}{216(1-\nu)},$$

Note that as before the spread of the monopolist is increasing in ν. These comparative statics are illustrated in Figure 9.

B.7 Closed form solutions and illustration

For the special case of uniform distributions, we can provide closed-form expressions for social surplus, as well as our measures of market thickness and the returns to scale in market making.

Corollary B.1. *If F and G are uniform distributions, then we have*

$$S_n^{FB}(\nu) = \frac{n}{(4n+2)(2n-1)}\left(n(2n-1) + (n(2n+1)-1)\nu + \sum_{i=2}^{2n} \nu^i\right).$$

Consequently, we have

$$T_n(\nu) = \frac{2(n-1)(2n-1) - 6(n-1)(2n+1)\nu^2 + 2(4n^2-1)\nu^3 - 6\nu^{2n+1}}{(4n^2-1)(2\nu+1)(1-\nu)^2}$$

and

$$\frac{T_{n+1}(\nu)}{T_n(\nu)} = \frac{(2n-1)\left[2(1-\nu)^2(2\nu+1)n^2 + 3\nu^3\left(1-\nu^{2n}\right) + n\left(8\nu^3 - 9\nu^2 + 1\right)\right]}{(2n+3)\left[1 + 3n\left(\nu^2-1\right) + 3\nu^2 - \nu^3 + 2n^2(\nu-1)^2(2\nu+1) - 3\nu^{2n+1}\right]}.$$

For mass products (i.e. setting $\nu = 0$), this yields

$$S_n^{FB}(0) = \frac{n^2}{4n+2}, \quad T_n(0) = \frac{2(n-1)}{2n+1} \quad \text{and} \quad \frac{T_{n+1}(0)}{T_n(0)} = \frac{n(2n+1)}{2n^2+n-3}.$$

Proof. Adopting the notation from the proof of Proposition 4 and combining

$$\begin{aligned} h_{\nu,n,n,n}(x) &= \sum_{i=0}^{n-1}(n-i)\binom{n}{i}\binom{n}{i} f_\nu(x)F_\nu(x)^i(1-G_\nu(x))^i(1-F_\nu(x))^{n-i-1}G_\nu(x)^{n-i} \\ &+ \sum_{i=0}^{n-1}(n-i)\binom{n}{i}\binom{n}{i} g_\nu(x)(1-F_\nu(x))^i G_\nu(x)^i F_\nu(x)^{n-i}(1-G_\nu(x))^{-i+n-1} \end{aligned}$$

and

$$\begin{aligned} & h_{\nu,n+1,n,n+1}(x) \\ &= \sum_{i=0}^{n}(n-i+1)\binom{n+1}{i}\binom{n}{i} f_\nu(x)F_\nu(x)^i(1-G_\nu(x))^i(1-F_\nu(x))^{n-i}G_\nu(x)^{n-i} \\ &+ \sum_{i=0}^{n-1}(n-i)\binom{n}{i}\binom{n+1}{i+1} g_\nu(x)(1-F_\nu(x))^{i+1}G_\nu(x)^i F_\nu(x)^{n-i}(1-G_\nu(x))^{-i+n-1}. \end{aligned}$$

with the results of Proposition 4 yields the desired expressions. □